

**STATE VARIABLE ANALYSIS OF A BOILER SYSTEM**

**Chusakdi Senanikrom**



# NAVAL POSTGRADUATE SCHOOL

## Monterey, California



# THESIS

State Variable Analysis of a Boiler System

by

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March 1978

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STATE VARIABLE ANALYSIS OF A BOILER SYSTEM

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Submitted in partial fulfillment of the  
requirements for the degree of

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## ABSTRACT

The state variable formulation of a Foster Wheeler ESD-III boiler is developed from fundamental principles. The response of the model for various input signals is determined using CSMP -III, the IBM simulation language. The sensitivity of the model to various coefficient values is noted as are the characteristics of various system states for small perturbation values.



## TABLE OF CONTENTS

I.	INTRODUCTION.....	12
II.	BOILER MODEL.....	13
A.	BOILER CONSIDERATIONS.....	13
B.	BOILER ANALYSIS.....	16
1.	Superheater.....	16
2.	Downcomer-riser loop.....	16
3.	Drum.....	17
4.	Gas path.....	17
5.	Superheater continuity.....	20
6.	Superheater energy balance.....	20
7.	Heat conduction across superheater walls.....	21
8.	Heat input to superheater wall.....	21
9.	Heat transfer across superheater wall.....	22
10.	Heat transfer from combustion gas.....	22
11.	Superheater momentum.....	23
12.	Riser continuity equation.....	25
13.	Riser momentum equation.....	26
14.	Heat balance across riser tube wall.....	27
15.	Riser heat balance equation.....	28
16.	Riser heat transfer equation.....	29
17.	Riser heat transfer to boiling fluid.....	30
18.	Downcomer momentum equation.....	30
19.	Drum steam mass balance equation.....	31
20.	Drum liquid heat balance equation.....	32
21.	Drum liquid mass balance equation.....	33
22.	Equation from throttle valve.....	34
23.	State equations.....	34
C.	STATE VARIABLE EQUATIONS.....	36
III.	COMPUTER RESULTS.....	51
A.	TRANSIENT RESPONSES FOLLOWING A STEP CHANGE IN	



THROTTLE SETTING OF 5%.....	52
B.    TRANSIENT RESPONSES FOLLOWING A STEP CHANGE OF 10% IN FUEL FLOW RATE.....	60
C.    TRANSIENT RESPONSES FOLLOWING A STEP CHANGE OF 10% IN FEEDWATER FLOW-RATE.....	68
IV.  CONCLUSIONS.....	76
Appendix A:  LIST OF COEFFICIENTS A.....	77
Appendix B:  LIST OF COEFFICIENTS B.....	84
Appendix C:  C MATRIX COEFFICIENTS.....	90
Appendix D:  D MATRIX COEFFICIENTS.....	93
Appendix E:  LIST OF CONSTANTS.....	94
Appendix F:  FORTRAN IV PROGRAM.....	97
Appendix G:  FORTRAN IV PROGRAM OUTPUT.....	102
Appendix H:  CSMP-III. PROGRAM.....	103
BIBLIOGRAPHY.....	105
INITIAL DISTRIBUTION LIST.....	106





## LIST OF FIGURES

1. A cross sectional arrangement of the Foster Wheeler D-type marine boiler.....	15
2. Schematic diagram of superheater.....	19
3. Schematic diagram of riser-downcomer loop.....	24
4. State variable matrix.....	50
5. Superheated steam density ( $\Delta\rho_s$ ) vs time and superheated steam temperature ( $\Delta T_s$ ) vs time.....	53
6. Superheater wall temperature ( $\Delta T_{sw}$ ) vs time and quality of mixture leaving riser ( $\Delta X$ ) vs time.....	54
7. Riser mixture flow rate ( $\Delta W$ ) vs time and downcomer liquid flow rate ( $\Delta W_w$ ) vs time.....	55
8. Riser tube wall temperature ( $\Delta T_{bw}$ ) vs time and steam drum pressure ( $\Delta P_g$ ) vs time.....	56
9. Steam drum liquid temperature ( $\Delta T_w$ ) vs time and steam drum water level ( $\Delta y$ ) vs time.....	57
10. Superheater outlet pressure ( $\Delta P_g$ ) vs time and steam flow rate at superheater outlet ( $\Delta W_s$ ) vs time.....	58
11. Steam mass-flow rate from steam drum to superheater ( $\Delta W_g$ ) vs time.....	59
12. Superheated steam density ( $\Delta\rho_s$ ) vs time and superheated steam temperature ( $\Delta T_s$ ) vs time.....	61
13. Superheater wall temperature ( $\Delta T_{sw}$ ) vs time and quality of mixture leaving riser ( $\Delta X$ ) vs time.....	62



14. Riser mixture flow rate ( $\Delta W$ ) vs time and downcomer liquid flow rate ( $\Delta W_w$ ) vs time.....	63
15. Riser tube wall temperature ( $\Delta T_{BW}$ ) vs time and steam drum pressure ( $\Delta P_B$ ) vs time.....	64
16. Steam drum liquid temperature ( $\Delta T_w$ ) vs time and steam drum water level ( $\Delta y$ ) vs time.....	65
17. Superheater outlet pressure ( $\Delta P_s$ ) vs time and steam flow rate at superheater outlet ( $\Delta W_s$ ) vs time.....	66
18. Steam mass-flow rate from steam drum to superheater( $\Delta W_8$ ) vs time.....	67
19. Superheated steam density ( $\Delta \rho_s$ ) vs time and superheated steam temperature ( $\Delta T_s$ ) vs time.....	69
20. Superheater wall temperature ( $\Delta T_{sw}$ ) vs time and quality of mixture leaving riser ( $\Delta X$ ) vs time.....	70
21. Riser mixture flow rate ( $\Delta W$ ) vs time and downcomer liquid flow rate ( $\Delta W_w$ ) vs time.....	71
22. Riser tube wall temperature ( $\Delta T_{BW}$ ) vs time and steam drum pressure ( $\Delta P_B$ ) vs time.....	72
23. Steam drum liquid temperature ( $\Delta T_w$ ) vs time and steam drum water level ( $\Delta y$ ) vs time.....	73
24. Superheater outlet pressure ( $\Delta P_s$ ) vs time and steam flow rate at superheater outlet ( $\Delta W_s$ ) vs time.....	74
25. Steam mass-flow rate from steam drum to superheater( $\Delta W_8$ ) vs time.....	75



## NOMENCLATURE

- $A_s, A_r, A_d$  = superheater, riser and downcomer cross-sectional areas respectively ( $\text{ft}^2$ )
- $C$  = evaporation level proportionality constant ( $\text{ft-sec/lb}$ )
- $C_s, C_r, C_d$  = heat capacitance for superheater, riser and downcomer tubes respectively ( $\text{BTU/lb-}^\circ\text{R}$ )
- $C_g$  = average heat capacitance of combustion gases times air-fuel ratio ( $\text{BTU/lb-}^\circ\text{R}$ )
- $C_i$  = heat capacitance for feedwater ( $\text{BTU/lb-}^\circ\text{R}$ )
- $C_{ps}$  = average heat capacitance of superheated steam ( $\text{BTU/lb-}^\circ\text{R}$ )
- $f_s, f_r, f_d$  = friction coefficients for superheater, riser and downcomer tubes, respectively ( $\text{sec}^2/\text{ft}^5$ )
- $g$  = acceleration due to gravity ( $\text{ft/sec}^2$ )
- $h_g$  = enthalpy of saturated vapour corresponding to  $P_B$  ( $\text{BTU/lb}$ )
- $h_{wg}$  = enthalpy of saturated liquid corresponding to  $P_B$  ( $\text{BTU/lb}$ )
- $h_{wb}$  = enthalpy of liquid in mud-drum ( $\text{BTU/lb}$ )
- $h_w$  = enthalpy of drum and downcomer liquid ( $\text{BTU/lb}$ )
- $h_{fg}$  = enthalpy of evaporation corresponding to  $P_B$  ( $\text{BTU/lb}$ )
- $K_{as}$  = air cooling coefficient at superheater bank ( $\text{BTU/sec}$ )
- $K_{ar}$  = air cooling coefficient at riser bank ( $\text{BTU/sec}$ )
- $K_{gs}, K_s$  = heat-transfer coefficients from combustion gas to superheater tubes, and from superheater tubes to steam, respectively ( $\text{BTU/lb-}^\circ\text{R}$ )
- $K_{gb}, K_r, K_g$  = heat-transfer coefficients from combustion gas to



riser tubes and from riser tubes to boiling

liquid respectively  $(\text{BTU}/\text{lb}^{\circ}\text{R})$ ,  $(\text{BTU}/\text{lb}^{\circ}\text{R}^4)$ ,  $(\text{BTU}/\text{lb}^{\circ}\text{R})$

$K_e$  = evaporation rate constant of drum liquid  $(\text{lb}/\text{sec}^{\circ}\text{R})$

$K_T, K_B$  = constants for state equations of saturated steam

$(^{\circ}\text{R}\text{-ft}^2/\text{lb})$ ,  $(\text{ft})^{-1}$

$L_s, L_r, L_D$  = superheater, riser and downcomer tube lengths respectively (ft)

$M$  = mass of drum liquid (lb)

$M_s$  = mass of superheater tubes (lb)

$M_B$  = mass of riser tubes (lb)

$P_B$  = drum pressure  $(\text{lb}/\text{ft}^2)$

$P_s$  = superheater outlet pressure  $(\text{lb}/\text{ft}^2)$

$P_w$  = mud-drum pressure  $(\text{lb}/\text{ft}^2)$

$Q_s$  = heat-input rate from tube walls into the superheated steam (BTU/sec)

$Q_{gs}$  = heat-input rate from hot gasses into superheater tube walls (BTU/sec)

$Q_B$  = heat-input rate from riser tube walls into boiling liquid (BTU/sec)

$Q_{gB}$  = heat-input rate from hot gasses into riser tube walls (BTU/sec)

$T_i$  = feedwater temperature ( $^{\circ}\text{R}$ )

$T_B$  = saturation temperature corresponding to  $P_B$  ( $^{\circ}\text{R}$ )

$T_s$  = superheater outlet temperature ( $^{\circ}\text{R}$ )

$T_w$  = drum and downcomer liquid temperature ( $^{\circ}\text{R}$ )

$T_{sw}$  = superheater tube-wall temperature ( $^{\circ}\text{R}$ )

$T_{Bw}$  = riser tube-wall temperature ( $^{\circ}\text{R}$ )

$T_{gs}$  = average gas temperature at superheater banks ( $^{\circ}\text{R}$ )

$T_{gB}$  = average gas temperature at riser banks ( $^{\circ}\text{R}$ )

$T_c$  = combustion gas temperature entering superheater banks ( $^{\circ}\text{R}$ )

$V_B$  = volume of vapor phase in drum  $(\text{ft}^3)$





$V_r$  = velocity of riser mixture (ft/sec)  
 $V_w$  = velocity of downcomer water (ft/sec)  
 $V$  = total drum volume (ft<sup>3</sup>)  
 $W_s$  = steam mass-flow rate at the superheater outlet  
 (lb/sec)  
 $W_f$  = fuel mass-flow rate (lb/sec)  
 $W_i$  = feedwater mass-flow rate (lb/sec)  
 $W_w$  = downcomer mass-flow rate (lb/sec)  
 $W$  = riser mass-flow rate (lb/sec)  
 $W_a$  = air mass-flow rate from blower (lb/sec)  
 $W_{ac}$  = chemically correct +50% excess air rate (lb/sec)  
 $W_e$  = mass-evaporation rate from drum liquid surface  
 (lb/sec)  
 $W_B$  = steam mass-flow rate from drum into superheater  
 (lb/sec)  
 $X$  = quality of mixture leaving riser  
 $X_v$  = throttle opening (%)  
 $y$  = drum liquid level (ft)  
 $\rho_B$  = saturated vapor density corresponding to  $P_B$  (lb/ft<sup>3</sup>)  
 $\rho_s$  = superheater outlet density (lb/ft<sup>3</sup>)  
 $\rho_w$  = saturated liquid density corresponding to  $P_B$  (lb/ft<sup>3</sup>)  
 $\rho$  = density of liquid vapor mixture leaving riser  
 (lb/ft<sup>3</sup>)  
 $\eta$  = evaporation level (ft)



## I. INTRODUCTION

Boilers as understood by marine engineers are closed vessels containing water which by the application of heat is converted into steam at any designed pressure. This steam is then used for the production, through machinery, of useful work. A dynamic model of steam turbine machinery consists of a boiler model and a turbine model. The difficult part is the boiler, as a load change causes variations in some important properties such as boiler pressure, temperature and drum water level. The analysis in this paper will use Chien's dynamic analysis (1) of a boiler as a reference. Chien considered a naval boiler which for purposes of analysis was divided into four sections namely a superheater, a downcomer-riser loop, a drum and a gas path.

The principles of thermodynamics, heat-transfer and fluid mechanics were used to describe the dynamic behaviour corresponding to each section of the boiler and these were derived from equations of continuity, energy, heat-transfer, and momentum. The equations involve partial differentials as well as nonlinear terms. These equations were reduced to the ordinary linear equation form by applying small perturbation and difference equation techniques. Linear equations thus obtained were reduced to ten state variable equations and solved by digital computer techniques.

Since there is an increasing interest in boiler modelling, the objective of this thesis was to develop a comprehensive boiler simulation model in a form useful for modern control (i.e., multivariable control) analysis.



## II. BOILER MODEL

### A. BOILER CONSIDERATIONS

The control problems of high pressure boilers have become more and more critical, both from the operational and the economical points of view. A dynamic analysis is the method to be used for a control-system analysis. It consists of a complete understanding of the process to be controlled and the effects of physical and chemical changes. The analysis is not exact by any means but the results obtained should be in good qualitative agreement with actual tests. The major difficulty in boiler analysis is the fact that the whole system is very complex and contains numerous variables.

Chien(1) considered a naval boiler which for purposes of analysis was divided into four distinct sections namely a superheater, a downcomer-riser loop, a drum and a gas path. The detail of analysis is described in Part B of this section. The basic equations used in the analysis are those of continuity, energy (heat-transfer), momentum and the state equations. These equations involve partial differentiation as well as nonlinearities. Generally, the equations have the form

$$f(x, y, z, \dots) = 0$$

To eliminate the nonlinearities one uses perturbation theory, which effectively approximates the response of the



system to small signal changes about a chosen operating condition. Thus, the equation is perturbed about its steady state operating condition to give the linearized form.

Hence it can be written as

$$\frac{\partial f}{\partial x} \Delta x + \frac{\partial f}{\partial y} \Delta y + \frac{\partial f}{\partial z} \Delta z + \dots = 0$$

The perturbed variables are  $\Delta X, \Delta Y, \Delta Z$ , etc. and the partial differentials ( $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$ , etc.) that form the coefficients of the perturbed variables are evaluated at steady state operating conditions. This technique was also followed by Whalley (2) in modeling the same boiler plant.





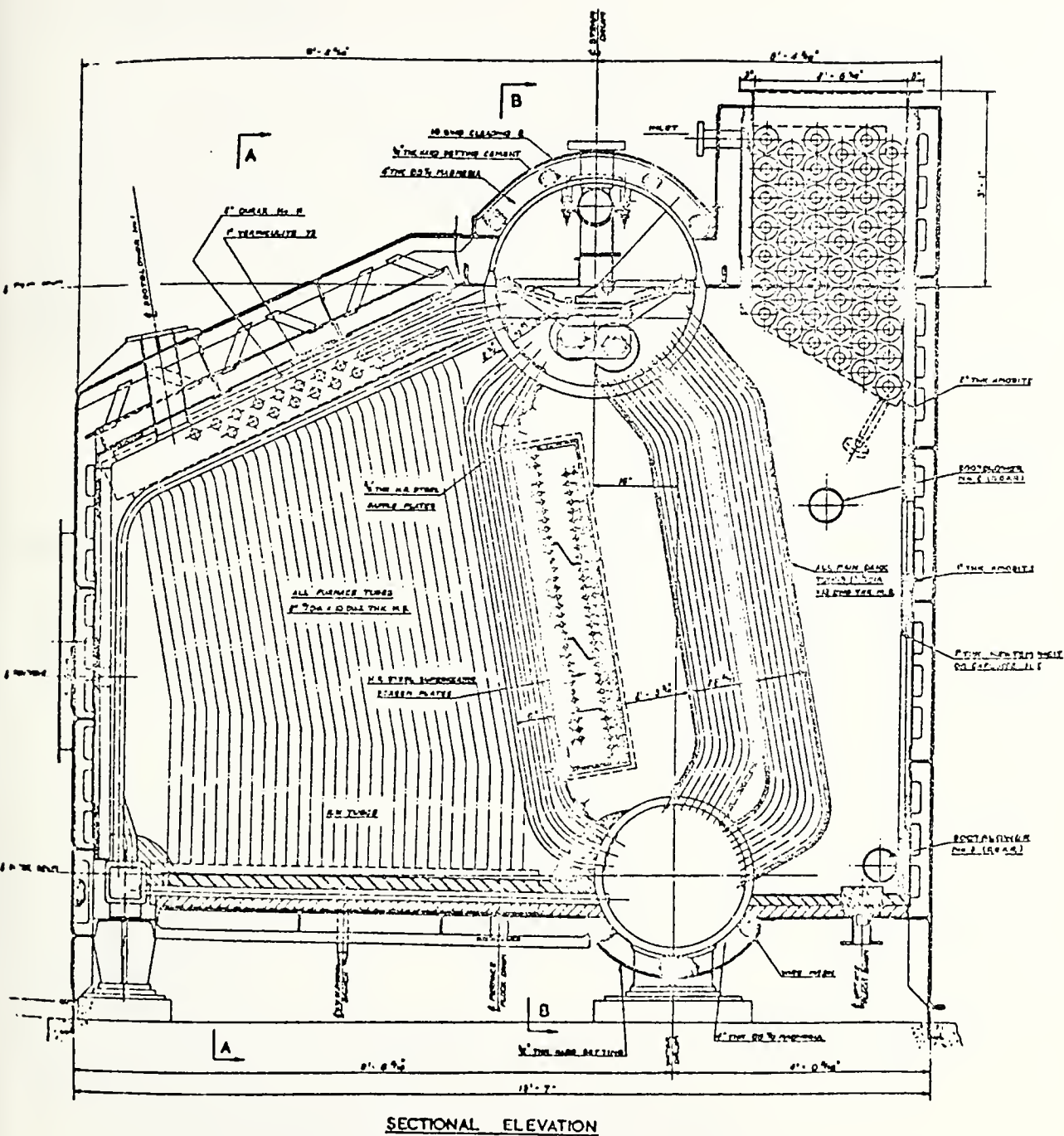


Figure 1 - A CROSS SECTIONAL ARRANGEMENT OF THE FOSTER  
WHEELER D-TYPE MARINE BOILER



## E. BOILER ANALYSIS

The boiler studied in this analysis is a Foster Wheeler D-type marine boiler; which is an oil fired, two-drum, natural circulation unit having a rated output of 28,800 lbs/hour at 350 lbs/in<sup>2</sup> gauge, 1200°F. A cross sectional arrangement of the unit is shown in Figure 1. The following assumptions apply to the physical simplifications in each of four sections.

### 1. Superheater

(a) The inertial effects of superheated steam are neglected.

(b) The superheater tubes are assumed to be a single capacitance with restriction on the drum side and another restriction on the load side.

(c) Desuperheaters are not considered.

### 2. Downcomer riser loop

(a) Only natural circulation exists.

(b) No boiling takes place in the downcomers.

(c) Vapor and liquid velocities in the riser are identical.

(d) Heat-transfer rates to the boiling liquid from the tube walls are proportional to the cube of temperature difference between the wall and the liquid.

(e) Steam quality is uniform in the riser.

(f) Liquid temperature is always the same as the saturation temperature corresponding to drum pressure.

(g) Downcomer liquid temperature is the same as the drum liquid temperature.



### 3. Drum

(a) There is no temperature gradient across the drum vapor phase, and the temperature is always the saturation temperature corresponding to drum pressure.

(b) The liquid phase has no temperature gradient other than across a very thin boundary layer at the drum surface.

(c) Evaporation or condensation rate in the drum is proportional to the difference of liquid and saturation temperatures.

(d) Feedwater temperature is assumed to be constant.

(e) Liquid-level changes due to bubble formation in the drum are neglected.

### 4. Gas Path

(a) The air-fuel ratio is assumed to be constant.

(b) Temperature of combustion gas entering superheater is proportional to the firing rate.

(c) Waterwalls are lumped with the riser-banks.

(d) The heat-transfer rate at each tube bank is determined by the tube wall temperature and the average gas temperature.

(e) Inertia of the hot gases is neglected, that is, velocity changes take place instantaneously.

(f) Delays due to the heat capacitance of the hot gases are neglected, that is, temperature changes take place instantaneously in combustion gases.

(g) All heat transfer is due to turbulent convection and radiation.

The following steps are taken in developing the equations for each of the four sections. A simple schematic diagram of each sub-section is included at the beginning of each part of the analysis. A brief statement of the physical



situation is included under the headings. The dummy coefficients such as  $a_1$ ,  $b_1$ ,  $C_{11}$ ,  $D_{11}$ , were used for convenience, and their values are declared in the Appendices. The quantities such as  $\bar{w}_B$ ,  $\bar{h}_S$ ,  $\bar{h}_B$ , etc. are the steady state values of  $w_B$ ,  $h_S$ ,  $h_B$ , respectively. These values can be evaluated from the original unperturbed nonlinear equations by setting all derivative terms to zero and solving for the unknown values in term of known values.





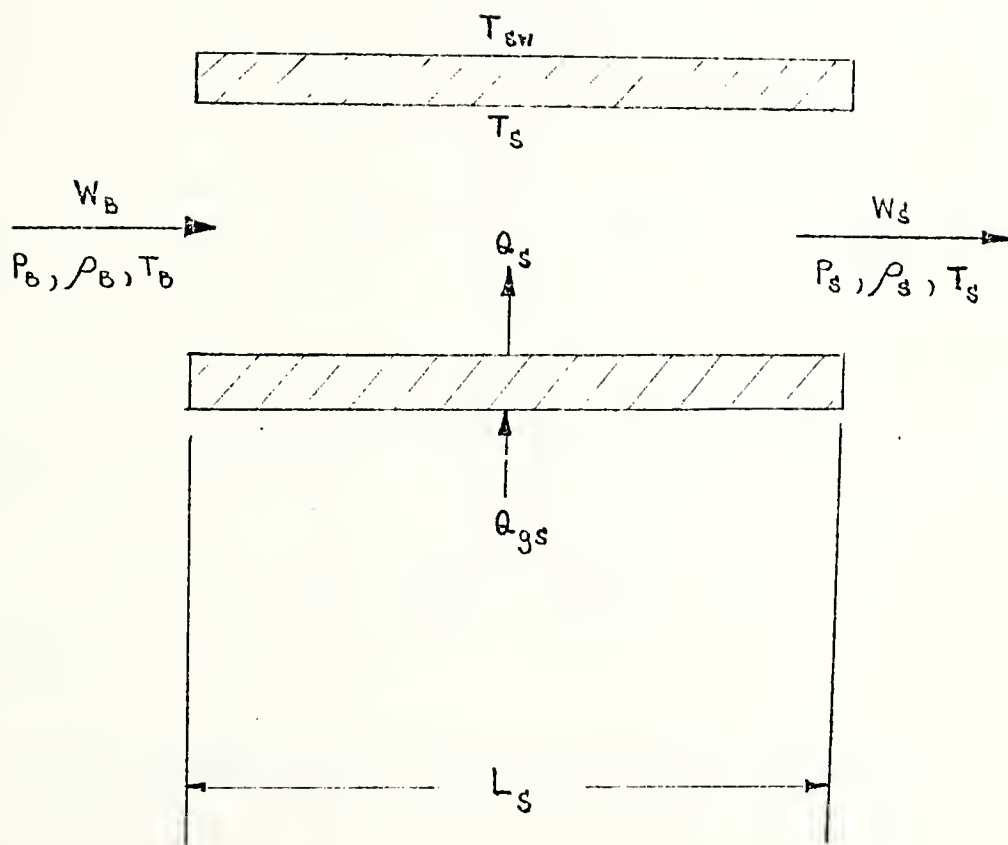


Figure 2 - SCHEMATIC DIAGRAM OF SUPERHEATER



### 5. Superheater continuity

Mass flow in - Mass flow out = Rate of change of enclosed mass

$$W_B - W_s = \frac{d}{dt}(L_s A_s \rho_s)$$

The perturbed equation is :

$$\Delta W_B - \Delta W_s = L_s A_s S \Delta \rho_s$$

or,

$$a_1 \Delta W_B + a_2 \Delta W_s = a_3 S \Delta \rho_s \quad (1)$$

where  $S$  is the Laplace variable denoting differentiation with respect to time

### 6. Superheater energy balance

Heat input from tube wall + Heat in incoming steam - Heat with outgoing steam = Rate of change of internal energy

$$Q_s + W_B h_B - W_s h_s = \frac{d}{dt}(L_s A_s \rho_s h_s)$$

The perturbed equation is :

$$\Delta Q_s + \bar{W}_B \Delta h_B + \bar{h}_B \Delta W_B - \bar{W}_s \Delta h_s - \bar{h}_s \Delta W_s = L_s A_s (\bar{\rho}_s S \Delta h_s + \bar{h}_s S \Delta \rho_s)$$

since,  $\Delta h_B = 0$ ,  $\Delta h_s = c_{ps} \Delta T_s$  and  $L_s A_s S \Delta \rho_s = \Delta W_B - \Delta W_s$

After substitution and rearrangement:

$$(L_s A_s \bar{\rho}_s c_{ps}) S \Delta T_s + (\bar{h}_s - \bar{h}_B) \Delta W_B - \Delta Q_s = -\bar{W}_s c_{ps} \Delta T_s \quad (2)$$

or,

$$a_4 S \Delta T_s + a_5 \Delta W_B + a_6 \Delta Q_s = a_7 \Delta T_s$$



### 7. Heat conduction across superheater walls

Heat input to wall - Heat output from wall = Rate of change of energy stored in superheater wall

$$Q_{gs} - Q_s = \frac{d}{dt} (M_s C_s T_{sw})$$

The perturbed equation is :

$$\Delta Q_{gs} - \Delta Q_s = M_s C_s S \Delta T_{sw}$$

or,

$$a_8 \Delta Q_{gs} + a_9 \Delta Q_s = a_{10} S \Delta T_{sw} \quad (3)$$

### 8. Heat input to superheater wall

The following empirical formula includes the cooling effect if air supply  $w_a$  differs from ideal air supply  $w_{ac}$  (Whalley (2)):

$$Q_{gs} = K_{gs} w_f^{0.6} (T_{gs} - T_{sw}) - K_{as} \left(1 - \frac{w_a}{w_{ac}}\right)^2$$

The perturbed equation is :

$$\Delta Q_{gs} = \frac{0.6 K_{gs}}{\bar{w}_f^{0.4}} (\bar{T}_{gs} - \bar{T}_{sw}) \Delta w_f + K_{gs} \bar{w}_f^{0.6} (\Delta T_{gs} - \Delta T_{sw}) + 2 K_{as} \frac{(\bar{w}_{ac} - \bar{w}_a)}{\bar{w}_{ac}^2} \Delta w_a$$

or,

$$a_{11} \Delta Q_{gs} = a_{12} \Delta w_f + a_{13} (\Delta T_{gs} - \Delta T_{sw}) + a_{14} \Delta w_a \quad (4)$$



### 9. Heat transfer across superheater wall

For turbulent gas flows:

$$Q_s = K_s W_B^{0.8} (T_{sw} - T_s)$$

The perturbed equation is :

$$\Delta Q_s = \frac{0.8}{W_B^{0.2}} K_s (\bar{T}_{sw} - \bar{T}_s) \Delta W_B + K_s \bar{W}_B^{0.8} (\Delta T_{sw} - \Delta T_s)$$

or,

$$a_{15} \Delta Q_s = a_{16} \Delta W_B + a_{17} (\Delta T_{sw} - \Delta T_s) \quad (5)$$

### 10. Heat transfer from combustion gas

Average gas temperature at superheater wall is  $T_{gs}$  where

$$T_{gs} = T_c - \frac{0.3 Q_{gs}}{C_g W_f}$$

and  $C_g$  is related to the heat capacitance of the combustion gas by :

$$C_g = C_c \left(1 + \frac{W_a}{W_f}\right)$$

where  $C_c$  is constant

Thus ;

$$Q_{gs} = 2 C_c (W_f + W_a) (T_c - T_{gs})$$

The perturbed equation is :

$$\Delta Q_{gs} = 2 C_c \left[ (\bar{W}_f + \bar{W}_a) \Delta T_c + (\bar{T}_c - \bar{T}_{gs}) \Delta W_f + (\bar{T}_c - \bar{T}_{gs}) \Delta W_a - (\bar{W}_f + \bar{W}_a) \Delta T_{gs} \right]$$





Rearranging:

$$\frac{1}{2C_c} \Delta Q_{gs} = (\bar{W}_f + \bar{W}_a) \Delta T_c + (\bar{T}_c - \bar{T}_{gs}) \Delta W_f + (\bar{T}_c - \bar{T}_{gs}) \Delta W_a - (\bar{W}_f + \bar{W}_a) \Delta T_{gs}$$

or,

$$a_{18} \Delta Q_{gs} = a_{19} \Delta T_c + a_{20} \Delta W_f + a_{21} \Delta W_a + a_{22} \Delta T_{gs} \quad (6)$$

### 11. Superheater momentum

Neglecting the inertia of the steam :

$$(P_B - P_S) A_s - \left( \frac{f L_s V_B^2}{g D_s} \right) A_s \rho_B = 0$$

Since :  $W_B = \rho_B A_s V_B$

$$(P_B - P_S) - \frac{f L_s}{g D_s} \left( \frac{W_B}{\rho_B A_s} \right)^2 \rho_B = 0$$

Substitute :  $f_s = \frac{f L_s}{g D_s A_s^2}$

The perturbed equation is :

$$\Delta P_B - \Delta P_S = 2 f_s \frac{\bar{W}_B}{\bar{\rho}_B} \Delta W_B - f_s \frac{\bar{W}_B^2}{\bar{\rho}_B^2} \Delta \rho_B$$

Rearranging :

$$2 f_s \frac{\bar{W}_B}{\bar{\rho}_B} \Delta W_B + \Delta P_S - f_s \frac{\bar{W}_B^2}{\bar{\rho}_B^2} \Delta \rho_B = \Delta P_B$$

or,

$$a_{23} \Delta W_B + a_{24} \Delta P_S + a_{25} \Delta \rho_B = a_{26} \Delta P_B \quad (7)$$



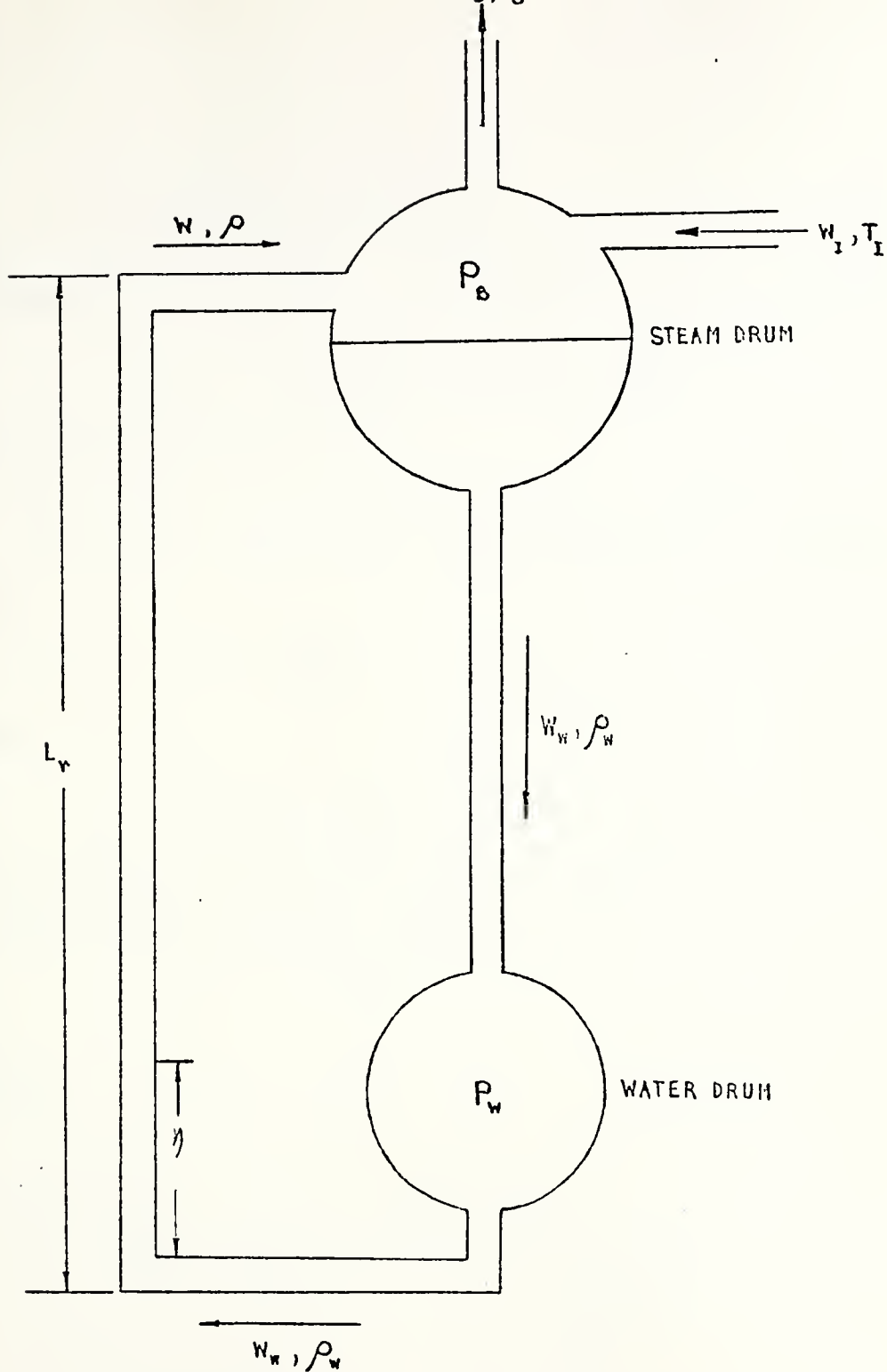


Figure 3 - SCHEMATIC DIAGRAM OF RISER-DOWNCOMER LOOP



## 12. Riser continuity equation

Mass flow into riser - Mass flow out = Rate of change of enclosed mass

$$W_w - W = \frac{d}{dt} \left[ A_r (L_r - \eta) (\rho + \rho_w) / 2 \right]$$

The perturbed equation is :

$$\Delta W_w - \Delta W = \frac{1}{2} \left[ A_r L_r S \Delta \rho - A_r (\rho + \rho_w) S \Delta \eta + A_r \bar{\eta} S \Delta \rho \right]$$

since

$$\frac{1}{\rho} = \frac{x}{\rho_B} + \frac{(1-x)}{\rho_w}$$

then,

$$\Delta \rho = \bar{\rho}^2 \left( \frac{1}{\rho_w} - \frac{1}{\rho_B} \right) \Delta x + \frac{\bar{\rho}^2 \bar{x}}{\bar{\rho}_B^2} \Delta \rho_B$$

If the starting point of evaporation in the riser tube is  $\eta$  above the water drum exit where :

$$\eta = 1 - C W_w$$

then,

$$\Delta \eta = -C \Delta W_w$$

$A_r \bar{\eta} S \Delta \rho$  can be neglected since  $\bar{\eta}$  is small  
Substituting  $\Delta \rho$  and  $\Delta \eta$  and rearranging :

$$0.5 A_r L_r \bar{\rho}^2 \left( \frac{1}{\rho_w} - \frac{1}{\rho_B} \right) S \Delta x + 0.5 A_r L_r \bar{x} K_B \left( \frac{\bar{\rho}^2}{\bar{\rho}_B^2} \right) S \Delta P_B$$

$$+ 0.5 (\bar{\rho} + \bar{\rho}_w) C S \Delta W_w = \Delta W_w - \Delta W$$

or,

$$a_{27} S \Delta x + a_{28} S \Delta P_B + a_{29} S \Delta W_w = a_{30} \Delta W_w + a_{31} \Delta W \quad (8)$$



### 13. Riser momentum equation

Pressure drop - frictional losses - gravity head - exit loss - momentum flux change = Inertia force

$$(P_W - P_B) A_r - \frac{f_r L_r W^2}{A_r^2 g D_r \rho^2} \cdot \rho A_r - A_r L_r \rho - \frac{W^2}{2 g (\rho A_r)^2} \cdot \rho A_r$$

$$- \left[ \frac{W^2}{g (\rho A_r)^2} \cdot \rho A_r - \frac{W_w^2}{g (\rho_w A_r)^2} \right] = \frac{L_r}{g} \frac{dw}{dt}$$

Therefore :

$$(P_W - P_B) - \left[ \frac{1}{A_r^2 g} \left( \frac{f_r L_r}{D_r} + \frac{3}{2} \right) \right] \frac{W^2}{\rho} + \frac{W_w^2}{g \rho_w A_r^2} - L_r \rho = \frac{L_r}{g A_r} \frac{dw}{dt}$$

After perturbation and substitution for  $\Delta \rho$  as done before in the Riser Continuity Equation, the perturbed equation is:

$$(\Delta P_W - \Delta P_B) - 2F \left[ \left( \frac{1}{\bar{\rho}_B} - \frac{1}{\bar{\rho}_W} \right) \bar{W} \bar{X} + \frac{\bar{W}}{\bar{\rho}_W} \right] \Delta W - F W^2 \left( \frac{1}{\bar{\rho}_B} - \frac{1}{\bar{\rho}_W} \right) \Delta X$$

$$+ \frac{F \bar{X} \bar{W}^2}{\bar{\rho}_B^2 \Delta \rho_B} - L_r \left[ \frac{\bar{X} \bar{\rho}^2}{\bar{\rho}_B^2 \Delta \rho_B} + \rho^2 \left( \frac{1}{\bar{\rho}_W} - \frac{1}{\bar{\rho}_B} \right) \right] \Delta X = \frac{L_r}{g A_r} S \Delta W$$

where :

$$F = \frac{\left( \frac{f_r L_r}{D_r} + \frac{3}{2} \right)}{A_r^2 g}$$

Rearranging :

$$\Delta P_W - \Delta P_B - (b_r + a_r S) \Delta W + b_X \Delta X - \frac{2 \bar{W}_w}{g A_r^2 \bar{\rho}_w} \Delta W_w + \infty \Delta \rho_B$$

where :

$$b_r = 2 \bar{W} \left( \frac{f_r L_r}{D_r} + \frac{3}{2} \right) / g A_r^2 \bar{\rho}$$

$$a_r = \frac{L_r}{g A_r}$$





$$b_x = \left( \frac{1}{\bar{\rho}_B} - \frac{1}{\bar{\rho}_W} \right) \left[ \frac{\bar{W}^2}{g A_r^2} \left( \frac{f_r L_r}{D_r} + \frac{3}{2} \right) - \bar{\rho}^2 L_r \right]$$

$$\alpha = - \frac{\bar{X}}{\bar{\rho}_B^2} \left[ \frac{\bar{W}^2}{A_r} \left( \frac{f_r L_r}{D_r} + \frac{3}{2} \right) + \bar{\rho}^2 L_r \right] \quad \text{and is small enough to be neglected}$$

Therefore :

$$\Delta P_W - \Delta P_B = b_v \Delta W + a_v S \Delta W + b_x \Delta X - \frac{2W_W}{g A_r^2 \bar{\rho}_W} \Delta W_W$$

or,

$$a_{32} \Delta P_W + a_{33} \Delta P_B = a_{34} \Delta W + a_{35} S \Delta W + a_{36} \Delta X + a_{37} \Delta W \quad (9)$$

#### 14. Heat balance across riser tube wall

Heat from hot gas to riser tubes - Heat from tubes to mixture  
= Rate of change of internal energy stored in tubes

$$Q_{gB} - Q_B = \frac{d}{dt} (M_B C_B T_{BW})$$

The perturbed equation is :

$$\Delta Q_{gB} - \Delta Q_B = M_B C_B S \Delta T_{BW}$$

or,

$$a_{38} \Delta Q_{gB} + a_{39} \Delta Q_B = a_{40} S \Delta T_{BW} \quad (10)$$



### 15. Riser heat balance equation

Heat input to risers - Heat output from risers

= Rate of change of internal energy

$$\begin{aligned} Q_B - W_w h_{wD} - W h &= \frac{d}{dt} \left[ 0.5 A_r (L_r - \eta) (\rho + \rho_w) h \right] \\ &= \bar{h} \frac{d}{dt} \left[ 0.5 A_r (L_r - \eta) (\rho + \rho_w) \right] + A_r (L_r - \eta) (\bar{\rho} + \bar{\rho}_w) \frac{d(h)}{dt} \end{aligned}$$

From the continuity equation :

$$\frac{d}{dt} \left[ 0.5 A_r (L_r - \eta) (\rho + \rho_w) \right] = W_w - W$$

After substitution and perturbation :

$$\Delta Q_B + \bar{W}_w \Delta h_{wD} + (\bar{h}_{wD} - \bar{h}) \Delta W_w - \bar{W} \Delta h = 0.5 A_r L_r (\rho + \rho_w) S \Delta h$$

since :

$$h = h_{wB} + X h_{fg}$$

If changes in latent heat  $\Delta h_{fg}$  are neglected, then

$$\Delta h = \Delta h_{wB} + \bar{h}_{fg} \Delta X$$

From steam tables :

$$\Delta h_{wB} = K_c \Delta P_B$$

$$\Delta h_{wD} = \Delta T_w$$

Therefore :

$$\begin{aligned} 0.5 A_r L_r (\bar{\rho} + \bar{\rho}_w) K_c S \Delta P_B + 0.5 A_r L_r (\bar{\rho} + \bar{\rho}_w) \bar{h}_{fg} S \Delta X - \Delta Q_B \\ = (\bar{h}_{wD} - \bar{h}) \Delta W_w - \bar{W} K_c \Delta P_B - \bar{W} \bar{h}_{fg} \Delta X + \bar{W}_w \Delta T_w \end{aligned}$$

or,

$$a_{41} S \Delta P_B + a_{42} S \Delta X + a_{43} \Delta Q_B = a_{44} \Delta W_w + a_{45} \Delta P_B + a_{46} \Delta X + a_{47} \Delta T_w \quad (11)$$



## 16. Riser heat transfer equation

An empirical formula that accounts for turbulent gas flow is adopted and gives :

$$Q_{gs} = K_{gs} W_f^{0.6} (T_{gs} - T_{BW}) - K_{ar} \left(1 - \frac{W_a}{W_{ac}}\right)^2 + K_r (T_{gs}^4 - T_{BW}^4)$$

The perturbed equation is :

$$\begin{aligned} \Delta Q_{gs} = & \frac{0.6 K_{gs} (\bar{T}_{gs} - \bar{T}_{BW})}{\bar{W}_f^{0.4}} \Delta W_f + K_{gs} \bar{W}_f (\Delta T_{gs} - \Delta T_{BW}) \\ & + \frac{2 K_{ar} (\bar{W}_{ac} - \bar{W}_a)}{\bar{W}_{ac}^2} \Delta W_a + 4 K_r (\bar{T}_{gs}^3 \Delta T_{gs} - \bar{T}_{BW}^3 \Delta T_{BW}) \end{aligned}$$

Rearranging :

$$\begin{aligned} \Delta Q_{gs} - K_{gs} \bar{W}_f^{0.6} \Delta T_{gs} = & - K_{gs} \bar{W}_f^{0.6} \Delta T_{BW} + \frac{0.6 K_{gs} (\bar{T}_{gs} - \bar{T}_{BW})}{\bar{W}_f^{0.4}} \Delta W_f \\ & + \frac{2 K_{ar} (\bar{W}_{ac} - \bar{W}_a)}{\bar{W}_{ac}^2} \Delta W_a + 4 K_r \bar{T}_{gs}^3 \Delta T_{gs} \\ & - 4 K_r \bar{T}_{BW}^3 \Delta T_{BW} \end{aligned}$$

hence :

$$\begin{aligned} \Delta Q_{gs} - (K_{gs} \bar{W}_f^{0.6} + 4 K_r \bar{T}_{gs}^3) \Delta T_{gs} = & - (K_{gs} \bar{W}_f^{0.6} + 4 K_r \bar{T}_{BW}^3) \Delta T_{BW} \\ & + \frac{0.6 K_{gs} (\bar{T}_{gs} - \bar{T}_{BW})}{\bar{W}_f^{0.4}} \Delta W_f \\ & + \frac{2 K_{ar} (\bar{W}_{ac} - \bar{W}_a)}{\bar{W}_{ac}^2} \Delta W_a \end{aligned}$$

or,

$$a_{48} \Delta Q_{gs} + a_{49} \Delta T_{gs} = a_{50} \Delta T_{BW} + a_{51} \Delta W_f + a_{52} \Delta W_a \quad (12)$$



### 17. Riser heat transfer to boiling fluid

The heat transfer rate is assumed to be proportional to the cube of temperature difference between wall and mixture.

Continuing to follow Whalley(2):

$$Q_B = K_g (\bar{T}_{BW} - T_B)^3$$

The perturbed equation is:

$$\Delta Q_B = 3 K_g (\bar{T}_{BW} - \bar{T}_B)^2 (\Delta T_{BW} - \Delta T_B)$$

or,

$$a_{53} \Delta Q_B = a_{54} \Delta T_{BW} + a_{55} \Delta T_B \quad (13)$$

### 18. Downcomer momentum equation

$\Sigma$  Forces due to pressure difference-frictional loss  
+gravitational head-entrance loss = Inertia force

Hence:

$$\begin{aligned} (P_B - P_W) A_D - \frac{f_D L_D}{g D_D} (V_W)^2 \rho_W A_D + \rho_W A_D (L_D + 2) - \frac{(V_W)^2}{2g} \rho_W A_D \\ = \frac{d}{dt} \frac{(L_D A_D \rho_W V_W)}{g} \end{aligned}$$

since:  $W_W = \rho_W A_D V_W$

$$(P_B - P_W) - \frac{f_D L_D W_W^2}{g \rho_W D_D A_D^2} + \rho_W L_D - \frac{W_W^2}{2g A_D \rho_W} = \frac{d}{dt} \frac{L_D}{g A_D} W_W$$

$$(P_B - P_W) - \left[ \left( \frac{f_D L_D}{D_D} + 0.5 \right) \frac{1}{g \rho_W A_D^2} \right] W_W^2 + \rho_W L_D = \frac{L_D}{g A_D} \frac{d}{dt} (W_W)$$





The perturbed equation is :

$$\Delta P_B - \Delta P_W = (b_W + a_W S) \Delta W_W - L_D \Delta \rho_W$$

where :

$$b_W = \frac{2 \bar{W}_W}{g \bar{\rho}_W A_D^2} \left( \frac{f_D L_D}{D_D} + 0.5 \right)$$

$$a_W = \frac{L_D}{g A_D}$$

Since flow is incompressible ( $\rho_W$  is constant both in time and space), then  $\Delta \rho_W = 0$

Therefore :

$$a_{56} \Delta P_B + a_{67} \Delta P_W = a_{58} \Delta W_W + a_{59} S \Delta W_W \quad (14)$$

#### 19. Drum steam mass balance equation

$\Sigma$  Vapor rates to/from drum = Rate of change of vapor mass in drum

Thus ,

Evaporation rate + vapor rate from risers-steam

$$\text{rate from drum} = \frac{d}{dt} (V_B \rho_B)$$

Therefore :

$$W_e + X W - W_B = \frac{d}{dt} (V_B \rho_B)$$

The perturbed equation is :

$$\Delta W_e + \bar{X} \Delta W + \bar{W} \Delta X - \Delta W_B = \bar{V}_B S \Delta \rho_B + \bar{\rho}_B S \Delta V_B$$

Substituting for :

$$\Delta V_B = -A \Delta y$$

$$\Delta \rho_B = K_B \Delta P_B$$

$$\Delta W_e = K_e (\Delta T_W - \Delta T_B)$$



gives :

$$\bar{V}_B K_B S \Delta P_B - \bar{f}_B A S \Delta y = K_c (\Delta T_w - \Delta T_B) + \bar{X} \Delta W + \bar{W} \Delta X - \Delta W_B$$

or,

$$a_{60} S \Delta P_B + a_{61} S \Delta y = a_{62} (\Delta T_w - \Delta T_B) + a_{63} \Delta W + a_{64} \Delta X + a_{65} \Delta W_B \quad (15)$$

## 20. Drum liquid heat balance equation

$\Sigma$  Heat to and from drum with liquid = Rate of change of the internal energy of the liquid

Hence :

Internal energy of riser liquid + heat transfered in feedwater - heat taken in by downcomer liquid - latent heat released with evaporating liquid =  $\frac{d}{dt}$  (drum liquid internal energy)

Therefore :

$$C_v (1-X) W T_B + C_i W_i T_i - C_D W_w T_w - W_e h_{BW} = \frac{d}{dt} (C_D M T_w)$$

But  $C_v, C_i, C_D = 1$ .

The perturbed equation is :

$$(1-\bar{X}) \bar{T}_B \Delta W + (1-\bar{X}) \bar{W} \Delta T_B - \bar{W} \bar{T}_B \Delta X + \bar{W}_i \Delta T_i + \bar{T}_i \Delta W_i - \bar{W}_w \Delta T_w$$

$$- \bar{T}_w \Delta W_w - \bar{W}_e \Delta h_{BW} - \bar{h}_{BW} \Delta W_e = \bar{M} S \Delta T_w + \bar{T}_w S \Delta M$$

Substituting for :

$$\Delta W_e = K_c (\Delta T_w - \Delta T_B)$$

$$\Delta h_{BW} = K_c \Delta P_B$$

$$\Delta M = A \rho_w \Delta y$$

and  $T_i \approx \text{constant}$ , so  $\Delta T_i \approx 0$



Thus:

$$(1-\bar{X})\bar{T}_B \Delta W + (1-\bar{X})\bar{W} \Delta T_B - \bar{W}\bar{T}_B \Delta X + \bar{T}_i \Delta W_i - \bar{W}_w \Delta T_w - \bar{T}_w \Delta W_w - \bar{W}_e K_e \Delta P_B$$

$$- \bar{h}_{BW} K_e (\Delta T_w - \Delta T_B) = \bar{M} S \Delta T_w + A \bar{\rho}_w \bar{T}_w S \Delta \gamma$$

or,

$$a_{66} \Delta W + a_{67} \Delta T_B + a_{68} \Delta X + a_{69} \Delta W_i + a_{70} \Delta T_w + a_{71} \Delta W_w + a_{72} \Delta P_B$$

$$+ a_{73} (\Delta T_w - \Delta T_B) = a_{74} S \Delta T_w + a_{75} S \Delta \gamma \quad (16)$$

## 21. Drum liquid mass balance equation

$\Sigma$  Liquid input/output rates = Rate of change enclosed mass of liquid

Hence:

$$\begin{aligned} &\text{feedwater input rate} + \text{riser liquid input rate} - \text{downcomer} \\ &\text{liquid rate} - \text{evaporation rate} = \frac{d}{dt} (M) \end{aligned}$$

Therefore:

$$W_i + (1-X)W - W_w - W_e = \frac{d}{dt} (M)$$

The perturbed equation is:

$$\Delta W_i + (1-\bar{X})\Delta W - \bar{W}\Delta X - \Delta W_w - \Delta W_e = S \Delta M$$

Substituting for:

$$\Delta W_e = K_e (\Delta T_w - \Delta T_B)$$

$$\Delta M = A \bar{\rho}_w \Delta \gamma$$

gives:

$$-K_e \Delta T_B + A \bar{\rho}_w S \Delta \gamma = (1-\bar{X})\Delta W - \bar{W}\Delta X - \Delta W_w - K_e \Delta T_w + \Delta W_i$$

or,

$$a_{66} \Delta T_B + a_{77} S \Delta \gamma = a_{83} \Delta W + a_{89} \Delta X + a_{90} \Delta W_w + a_{91} \Delta T_w + a_{92} \Delta W_i \quad (17)$$



22. Equation from throttle valve

$$W_s = f_1 (x_v, p_s, T_s)$$

The perturbed equation is :

$$\Delta W_s = \frac{\partial f_1}{\partial x} \Delta x_v + \frac{\partial f_1}{\partial p_s} \Delta p_s + \frac{\partial f_1}{\partial T_s} \Delta T_s$$

or,

$$a_{93} \Delta W_s = a_{94} \Delta x_v + a_{95} \Delta p_s + a_{96} \Delta T_s \quad (18)$$

23. State equations

$$\Delta T_B = \frac{\partial T_B}{\partial p_B} \Delta p_B = K_T \Delta p_B$$

or,

$$\Delta T_B = a_{97} \Delta p_B \quad (19)$$

$$\Delta p_B = \frac{\partial p_B}{\partial p_B} \Delta p_B = K_B \Delta p_B$$

or,

$$\Delta p_B = a_{98} \Delta p_B \quad (20)$$

$$\Delta T_{gs} = \Delta T_{g3} \quad (21)$$

$$\Delta h_{BW} = \frac{\partial h_{BW}}{\partial p_B} \Delta p_B = K_C \Delta p_B$$





or,

$$\Delta h_{BW} = a_{99} \Delta P_B \quad (22)$$

$$P_s = f_2(T_s, \rho_s)$$

$$\Delta P_s = \frac{\partial f_2}{\partial T_s} \Delta T_s + \frac{\partial f_2}{\partial \rho_s} \Delta \rho_s$$

or,

$$\Delta P_s = a_{100} \Delta T_s + a_{101} \Delta \rho_s \quad (23)$$

$$\Delta T_c = K_f \Delta W_f$$

or,

$$\Delta T_c = a_{102} \Delta W_f \quad (24)$$



### C. STATE VARIABLE EQUATIONS

The following procedure is an algebraic method to reduce the twenty-four governing equations to ten state variable equations, that is, in standard matrix form of  $\dot{X}(t) = AX(t) + BU(t)$

Here  $X(t)$  is the matrix of system state variables and  $U(t)$  is the matrix of system inputs.

From (1)

$$\begin{aligned} a_3 \dot{\Delta \rho}_s &= a_1 \Delta W_B + a_2 \Delta W_S \\ \dot{\Delta \rho}_s &= \frac{a_1}{a_3} \Delta W_B + \frac{a_2}{a_3} \Delta W_S \end{aligned}$$

or,

$$\dot{\Delta \rho}_s = b_1 \Delta W_B + b_2 \Delta W_S \quad (1.1)$$

From (2)

$$\begin{aligned} a_4 \Delta \ddot{T}_s &= -a_5 \Delta W_B - a_6 \Delta Q_s + a_7 \Delta T_s \\ \Delta \ddot{T}_s &= -\frac{a_5}{a_4} \Delta W_B - \frac{a_6}{a_4} \Delta Q_s + \frac{a_7}{a_4} \Delta T_s \end{aligned} \quad (1.2)$$

From (5)

$$\Delta Q_s = \frac{a_{16}}{a_{15}} \Delta W_B + \frac{a_{17}}{a_{15}} \Delta T_{sw} - \frac{a_{17}}{a_{15}} \Delta T_s \quad (1.3)$$

Substituting  $Q_s$  from (1.3) in (1.2) and rearranging :

$$\dot{\Delta T}_s = \left( \frac{a_{16} \cdot a_{17}}{a_4 \cdot a_{15}} + \frac{a_7}{a_4} \right) \Delta T_s - \left( \frac{a_5}{a_4} + \frac{a_{16} \cdot a_{16}}{a_4 \cdot a_{15}} \right) \Delta W_B - \frac{a_6 \cdot a_{17}}{a_4 \cdot a_{15}} \Delta T_{sw}$$



or,

$$\Delta \dot{T}_s = b_3 \Delta T_s - b_4 \Delta W_B - b_5 \Delta T_{sw} \quad (1.4)$$

From (3)

$$\begin{aligned} a_{10} S \Delta T_{sw} &= a_8 \Delta Q_{gs} + a_9 \Delta Q_s \\ \Delta \dot{T}_{sw} &= \frac{a_8}{a_{10}} \Delta Q_{gs} + \frac{a_9}{a_{10}} \Delta Q_s \end{aligned} \quad (1.5)$$

From (4)

$$\Delta Q_{gs} = \frac{a_{12}}{a_{11}} \Delta W_f + \frac{a_{13}}{a_{11}} \Delta T_{gs} - \frac{a_{13}}{a_{11}} \Delta T_{sw} + a_{14} \Delta W_a \quad (1.6)$$

From (6)

$$\Delta Q_{gs} = \frac{a_{19}}{a_{18}} \Delta T_c + \frac{a_{20}}{a_{18}} \Delta W_f + \frac{a_{21}}{a_{18}} \Delta W_a + \frac{a_{22}}{a_{18}} \Delta T_{gs} \quad (1.7)$$

Setting (1.6) = (1.7) and rearranging:

$$\begin{aligned} \Delta T_{gs} = & - \frac{\left( \frac{a_{12}}{a_{11}} - \frac{a_{20}}{a_{18}} \right)}{\left( \frac{a_{13}}{a_{11}} - \frac{a_{22}}{a_{18}} \right)} \Delta W_f + \frac{\left( \frac{a_{13}}{a_{11}} \right)}{\left( \frac{a_{13}}{a_{11}} - \frac{a_{22}}{a_{18}} \right)} \Delta T_{sw} \\ & - \frac{\left( a_{14} - \frac{a_{21}}{a_{18}} \right)}{\left( \frac{a_{13}}{a_{11}} - \frac{a_{22}}{a_{18}} \right)} \Delta W_a + \frac{\left( \frac{a_{19}}{a_{18}} \right)}{\left( \frac{a_{13}}{a_{11}} - \frac{a_{22}}{a_{18}} \right)} \Delta T_c \end{aligned} \quad (1.8)$$

Substituting  $\Delta T_c = a_{102} \Delta W_f$  from (24) into (1.8) and rearranging:

$$\Delta T_{gs} = \frac{\left( \frac{a_{19} \cdot a_{102}}{a_{18}} - \frac{a_{12}}{a_{11}} + \frac{a_{20}}{a_{18}} \right)}{\left( \frac{a_{13}}{a_{11}} - \frac{a_{22}}{a_{18}} \right)} \Delta W_f + \frac{\left( \frac{a_{13}}{a_{11}} \right)}{\left( \frac{a_{13}}{a_{11}} - \frac{a_{22}}{a_{18}} \right)} \Delta T_{sw} - \frac{\left( a_{14} - \frac{a_{21}}{a_{18}} \right)}{\left( \frac{a_{13}}{a_{11}} - \frac{a_{22}}{a_{18}} \right)} \Delta W_a$$



or,

$$\Delta T_{gs} = b_6 \Delta W_f + b_7 \Delta T_{sw} - b_8 \Delta W_a \quad (1.9)$$

Substituting  $\Delta T_{gs}$  from (1.9) into (1.6) and rearranging :

$$\Delta Q_{gs} = \left( \frac{a_{12}}{a_{11}} + \frac{a_{13} \cdot b_6}{a_{11}} \right) \Delta W_f + \left( \frac{b_7 \cdot a_{13}}{a_{11}} - \frac{a_{13}}{a_{11}} \right) \Delta T_{sw} + \left( a_{14} - \frac{b_8 \cdot a_{13}}{a_{11}} \right) \Delta W_a$$

or,

$$\Delta Q_{gs} = b_9 \Delta W_f + b_{10} \Delta T_{sw} + b_{11} \Delta W_a \quad (1.10)$$

Substituting  $\Delta Q_{gs}$  from (1.10) and  $\Delta Q_s$  from (1.3) into (1.5) and rearranging :

$$\begin{aligned} \Delta T_{sw} = & \left( \frac{a_8 \cdot b_9}{a_{10}} \right) \Delta W_f + \left( \frac{a_8 \cdot b_{10}}{a_{10}} + \frac{a_9 \cdot a_{17}}{a_{10} \cdot a_{15}} \right) \Delta T_{sw} + \left( \frac{a_8 \cdot b_{11}}{a_{10}} \right) \Delta W_a \\ & + \left( \frac{a_9 \cdot a_{16}}{a_{10} \cdot a_{15}} \right) \Delta W_B - \left( \frac{a_9 \cdot a_{17}}{a_{10} \cdot a_{15}} \right) \Delta T_s \end{aligned}$$

or,

$$\Delta T_{sw} = b_{12} \Delta W_f + b_{13} \Delta T_{sw} + b_{14} \Delta W_a + b_{15} \Delta W_B - b_{16} \Delta T_s \quad (1.11)$$

From (17)

$$\begin{aligned} \Delta \dot{y} = & - \frac{a_{76}}{a_{77}} \Delta T_B + \frac{a_{83}}{a_{77}} \Delta W + \frac{a_{89}}{a_{77}} \Delta X \\ & + \frac{a_{90}}{a_{77}} \Delta W_w + \frac{a_{91}}{a_{77}} \Delta T_w + \frac{a_{92}}{a_{77}} \Delta W_i \end{aligned} \quad (1.12)$$





Substituting  $\Delta T_B$  from (19) into (1.12)

$$\begin{aligned}\Delta \dot{Y} = & - \frac{a_{76} \cdot a_{97}}{a_{77}} \Delta P_B + \frac{a_{88}}{a_{77}} \Delta W + \frac{a_{89}}{a_{77}} \Delta X \\ & + \frac{a_{90}}{a_{77}} \Delta W_W + \frac{a_{91}}{a_{77}} \Delta T_W + \frac{a_{92}}{a_{77}} \Delta W_i\end{aligned}$$

OR,

$$\Delta \dot{Y} = -b_{17} \Delta P_B + b_{18} \Delta W + b_{19} \Delta X + b_{20} \Delta W_W + b_{21} \Delta T_W + b_{22} \Delta W_i \quad (1.13)$$

From (16)

$$\begin{aligned}\Delta \dot{T}_W = & - \frac{a_{75}}{a_{74}} \Delta \dot{Y} + \frac{a_{66}}{a_{74}} \Delta W + \frac{(a_{67} - a_{73})}{a_{74}} \Delta T_B + \frac{a_{68}}{a_{74}} \Delta X \\ & + \frac{a_{69}}{a_{74}} \Delta W_i + \frac{(a_{70} + a_{73})}{a_{74}} \Delta T_W + \frac{a_{71}}{a_{74}} \Delta W_W + \frac{a_{72}}{a_{74}} \Delta P_B\end{aligned} \quad (1.14)$$

Substituting  $\Delta \dot{Y}$  from (1.13) and  $\Delta T_B$  from (19) into (1.14) and rearranging :

$$\begin{aligned}\Delta \dot{T}_W = & \left( \frac{a_{75} \cdot b_{17}}{a_{74}} + \frac{a_{97} \cdot a_{67}}{a_{74}} - \frac{a_{97} \cdot a_{73}}{a_{74}} \right) \Delta P_B + \left( \frac{a_{66}}{a_{74}} - \frac{a_{75} \cdot b_{18}}{a_{74}} \right) \Delta W \\ & + \left( \frac{a_{68}}{a_{74}} - \frac{a_{75} \cdot b_{19}}{a_{74}} \right) \Delta X + \left( \frac{a_{71}}{a_{74}} - \frac{a_{75} \cdot b_{20}}{a_{74}} \right) \Delta W_W \\ & + \left( \frac{a_{70} + a_{73}}{a_{74}} - \frac{a_{75} \cdot b_{21}}{a_{74}} \right) \Delta T_W + \left( \frac{a_{69}}{a_{74}} - \frac{a_{75} \cdot b_{22}}{a_{74}} \right) \Delta W_i\end{aligned}$$

OR,

$$\Delta \dot{T}_W = b_{23} \Delta P_B + b_{24} \Delta W + b_{25} \Delta X + b_{26} \Delta W_W + b_{27} \Delta T_W + b_{28} \Delta W_i \quad (1.15)$$



From (15)

$$\Delta \dot{P}_B = -\frac{a_{61}}{a_{60}} \Delta \dot{Y} + \frac{a_{62}}{a_{60}} \Delta T_W - \frac{a_{62}}{a_{60}} \Delta T_B + \frac{a_{63}}{a_{60}} \Delta W + \frac{a_{64}}{a_{60}} \Delta X + \frac{a_{65}}{a_{60}} \Delta W_B \quad (1.16)$$

Substituting  $\Delta \dot{Y}$  from (1.13) into (1.16) and rearranging :

$$\begin{aligned} \Delta \dot{P}_B = & \left( \frac{a_{61} \cdot b_{17}}{a_{60}} - \frac{a_{62} \cdot a_{97}}{a_{60}} \right) \Delta P_B + \left( \frac{a_{63}}{a_{60}} - \frac{a_{61} \cdot b_{18}}{a_{60}} \right) \Delta W + \left( \frac{a_{64}}{a_{60}} - \frac{a_{61} \cdot b_{19}}{a_{60}} \right) \Delta X \\ & - \left( \frac{a_{61} \cdot b_{20}}{a_{60}} \right) \Delta W_W + \left( \frac{a_{62}}{a_{60}} - \frac{a_{61} \cdot b_{21}}{a_{60}} \right) \Delta T_W \\ & - \left( \frac{a_{61} \cdot b_{22}}{a_{60}} \right) \Delta W_i + \left( \frac{a_{65}}{a_{60}} \right) \Delta W_B \end{aligned}$$

or,

$$\begin{aligned} \Delta \dot{P}_B = & b_{29} \Delta P_B + b_{30} \Delta W + b_{31} \Delta X - b_{32} \Delta W_W \\ & + b_{33} \Delta T_W - b_{34} \Delta W_i + b_{35} \Delta W_B \end{aligned} \quad (1.17)$$

From (11)

$$\begin{aligned} \Delta \dot{X} = & -\frac{a_{41}}{a_{42}} \Delta \dot{P}_B - \frac{a_{43}}{a_{42}} \Delta Q_B + \frac{a_{44}}{a_{42}} \Delta W_W + \frac{a_{45}}{a_{42}} \Delta P_B \\ & + \frac{a_{46}}{a_{42}} \Delta X + \frac{a_{47}}{a_{42}} \Delta T_W \end{aligned} \quad (1.18)$$

From (13)

$$\Delta Q_B = \frac{a_{54}}{a_{53}} \Delta T_{BW} + \frac{a_{55}}{a_{53}} \Delta T_B$$



Substituting  $\Delta T_B$  from (19)

$$\Delta Q_B = \frac{a_{54}}{a_{53}} \Delta T_{BW} + \frac{a_{55} \cdot a_{97}}{a_{53}} \Delta P_B \quad (1.19)$$

Substituting  $\Delta \dot{P}_B$  from (1.17) and  $\Delta Q_B$  from (1.19) into (1.18) and rearranging :

$$\begin{aligned} \Delta \dot{X} = & \left( \frac{a_{45}}{a_{42}} - \frac{a_{41} \cdot b_{29}}{a_{42}} - \frac{a_{43} \cdot a_{55} \cdot a_{97}}{a_{42} \cdot a_{53}} \right) \Delta P_B - \left( \frac{a_{41} \cdot b_{30}}{a_{42}} \right) \Delta W \\ & + \left( \frac{a_{46}}{a_{42}} - \frac{a_{41} \cdot b_{31}}{a_{42}} \right) \Delta X + \left( \frac{a_{44}}{a_{42}} + \frac{a_{41} \cdot b_{32}}{a_{42}} \right) \Delta W_W + \left( \frac{a_{47}}{a_{42}} - \frac{a_{41} \cdot b_{33}}{a_{42}} \right) \Delta T_W \\ & + \left( \frac{a_{41} \cdot b_{34}}{a_{42}} \right) \Delta W_i - \left( \frac{a_{41} \cdot b_{35}}{a_{42}} \right) \Delta W_B - \left( \frac{a_{43} \cdot a_{54}}{a_{42} \cdot a_{53}} \right) \Delta T_{BW} \end{aligned}$$

or,

$$\begin{aligned} \Delta \dot{X} = & b_{36} \Delta P_B - b_{37} \Delta W + b_{38} \Delta X + b_{39} \Delta W_W \\ & + b_{40} \Delta T_W + b_{41} \Delta W_i - b_{42} \Delta W_B - b_{43} \Delta T_{BW} \end{aligned} \quad (1.20)$$

From (8)

$$\Delta \dot{W} = - \frac{a_{27}}{a_{29}} \Delta \dot{X} - \frac{a_{28}}{a_{29}} \Delta \dot{P}_B + \frac{a_{30}}{a_{29}} \Delta W_W + \frac{a_{31}}{a_{29}} \Delta W \quad (1.21)$$

Substituting  $\Delta \dot{X}$  from (1.20) and  $\Delta \dot{P}_B$  from (1.17) into (1.21) and rearranging :



$$\begin{aligned}
\Delta \dot{W}_W = & - \left( \frac{a_{27} \cdot b_{36}}{a_{29}} + \frac{a_{28} \cdot b_{29}}{a_{29}} \right) \Delta P_B + \left( \frac{a_{27} \cdot b_{37}}{a_{29}} - \frac{a_{28} \cdot b_{30}}{a_{29}} + \frac{a_{31}}{a_{29}} \right) \Delta W \\
& - \left( \frac{a_{27} \cdot b_{38}}{a_{29}} + \frac{a_{28} \cdot b_{31}}{a_{29}} \right) \Delta X + \left( \frac{a_{28} \cdot b_{32}}{a_{29}} + \frac{a_{30}}{a_{29}} - \frac{a_{27} \cdot b_{39}}{a_{29}} \right) \Delta W_W \\
& - \left( \frac{a_{27} \cdot b_{40}}{a_{29}} + \frac{a_{28} \cdot b_{33}}{a_{29}} \right) \Delta T_W + \left( \frac{a_{28} \cdot b_{34}}{a_{29}} - \frac{a_{27} \cdot b_{41}}{a_{29}} \right) \Delta W_i \\
& + \left( \frac{a_{27} \cdot b_{42}}{a_{29}} - \frac{a_{28} \cdot b_{35}}{a_{29}} \right) \Delta W_B + \left( \frac{a_{27} \cdot b_{43}}{a_{29}} \right) \Delta T_{BW}
\end{aligned}$$

or,

$$\begin{aligned}
\Delta \dot{W}_W = & -b_{44} \Delta P_B + b_{45} \Delta W - b_{46} \Delta X + b_{47} \Delta W_W - b_{48} \Delta T_W \\
& + b_{49} \Delta W_i + b_{50} \Delta W_B + b_{51} \Delta T_{BW} \quad (1.22)
\end{aligned}$$

Substituting  $\Delta \dot{W}_W$  from (1.22) into (14) and rearranging :

$$\begin{aligned}
\Delta P_W = & \left( \frac{a_{58} + a_{59} \cdot b_{47}}{a_{57}} \right) \Delta W_W - \left( \frac{a_{56} + a_{59} \cdot b_{44}}{a_{57}} \right) \Delta P_B + \left( \frac{a_{59} \cdot b_{43}}{a_{57}} \right) \Delta W \\
& - \left( \frac{a_{59} \cdot b_{46}}{a_{57}} \right) \Delta X - \left( \frac{a_{59} \cdot b_{48}}{a_{57}} \right) \Delta T_W + \left( \frac{a_{59} \cdot b_{49}}{a_{57}} \right) \Delta W_i \\
& + \left( \frac{a_{59} \cdot b_{50}}{a_{57}} \right) \Delta W_B + \left( \frac{a_{59} \cdot b_{51}}{a_{57}} \right) \Delta T_{BW}
\end{aligned}$$





or,

$$\begin{aligned}\Delta P_W = & b_{52} \Delta W_W - b_{53} \Delta P_B + b_{54} \Delta W - b_{55} \Delta X - b_{56} \Delta T_W \\ & + b_{57} \Delta W_i + b_{58} \Delta W_B + b_{59} \Delta T_{BW}\end{aligned}\quad (1.23)$$

From (9)

$$\Delta \dot{W} = \frac{a_{32}}{a_{35}} \Delta P_W + \frac{a_{33}}{a_{35}} \Delta P_B - \frac{a_{34}}{a_{35}} \Delta W - \frac{a_{36}}{a_{35}} \Delta X - \frac{a_{37}}{a_{35}} \Delta W_W$$

Substituting  $\Delta P_W$  from (1.23) into (1.24) and rearranging :

$$\begin{aligned}\Delta \dot{W} = & \left( \frac{a_{32} \cdot b_{52}}{a_{35}} - \frac{a_{37}}{a_{35}} \right) \Delta W_W + \left( \frac{a_{33}}{a_{35}} - \frac{a_{32} \cdot b_{53}}{a_{35}} \right) \Delta P_B + \left( \frac{a_{32} \cdot b_{54}}{a_{35}} - \frac{a_{34}}{a_{35}} \right) \Delta W \\ & - \left( \frac{a_{32} \cdot b_{55}}{a_{35}} + \frac{a_{36}}{a_{35}} \right) \Delta X - \left( \frac{a_{32} \cdot b_{56}}{a_{35}} \right) \Delta T_W + \left( \frac{a_{32} \cdot b_{57}}{a_{35}} \right) \Delta W_i \\ & + \left( \frac{a_{32} \cdot b_{58}}{a_{35}} \right) \Delta W_B + \left( \frac{a_{32} \cdot b_{59}}{a_{35}} \right) \Delta T_{BW}\end{aligned}$$

or,

$$\begin{aligned}\Delta \dot{W} = & b_{60} \Delta W_W + b_{61} \Delta P_B + b_{62} \Delta W - b_{63} \Delta X - b_{64} \Delta T_W \\ & + b_{65} \Delta W_i + b_{66} \Delta W_B + b_{67} \Delta T_{BW}\end{aligned}\quad (1.25)$$

From (10)

$$\Delta T_{BW} = \frac{a_{38}}{a_{40}} \Delta Q_{yB} + \frac{a_{39}}{a_{40}} \Delta Q_B \quad (1.26)$$



From (12)

$$a_{48} \Delta Q_{gB} = a_{50} \Delta T_{BW} + a_{51} \Delta W_f + a_{52} \Delta W_a - a_{49} \Delta T_{gS} \quad (1.27)$$

Since  $\Delta T_{gB} = \Delta T_{gS}$  from (21), then substituting  $\Delta T_{gS}$

From (1.9) into (1.27) and rearranging ;

$$\begin{aligned} \Delta Q_{gB} &= \frac{a_{50}}{a_{48}} \Delta T_{BW} + \left( \frac{a_{51} - a_{49} \cdot b_6}{a_{48}} \right) \Delta W_f + \left( \frac{a_{52} + a_{49} \cdot b_3}{a_{48}} \right) \Delta W_a \\ &\quad - \left( \frac{a_{49} \cdot b_7}{a_{48}} \right) \Delta T_{SW} \end{aligned} \quad (1.28)$$

Substituting  $\Delta Q_{gB}$  from (1.28) and  $\Delta Q_B$  from (1.19) into (1.26) and rearranging ;

$$\begin{aligned} \Delta \dot{T}_{BW} &= \left( \frac{a_{38} \cdot a_{50}}{a_{40} \cdot a_{48}} + \frac{a_{39} \cdot a_{54}}{a_{40} \cdot a_{53}} \right) \Delta T_{BW} + \left( \frac{a_{33}}{a_{40} \cdot a_{49}} \right) \left( a_{51} - a_{49} \cdot b_6 \right) \Delta W_f \\ &\quad + \left( \frac{a_{33}}{a_{40} \cdot a_{48}} \right) \left( a_{52} + a_{49} \cdot b_3 \right) \Delta W_a - \left( \frac{a_{33} \cdot a_{49} \cdot b_7}{a_{40} \cdot a_{48}} \right) \Delta T_{SW} \\ &\quad + \left( \frac{a_{39} \cdot a_{55} \cdot a_{97}}{a_{40} \cdot a_{53}} \right) \Delta P_B \end{aligned}$$

or,

$$\Delta \dot{T}_{BW} = b_{68} \Delta T_{BW} + b_{69} \Delta W_f + b_{70} \Delta W_a - b_{71} \Delta T_{SW} + b_{72} \Delta P_B \quad (1.29)$$

From (7)

$$\Delta W_B = \frac{a_{26}}{a_{23}} \Delta P_B - \frac{a_{24}}{a_{23}} \Delta P_S - \frac{a_{25}}{a_{23}} \Delta P_8 \quad (1.30)$$



Substituting  $\Delta P_s$  from (23) and  $\Delta \rho_s$  from (20) into (1.30)

$$\Delta W_B = \left( \frac{a_{26}}{a_{23}} - \frac{a_{25} \cdot a_{98}}{a_{23}} \right) \Delta P_B - \left( \frac{a_{24} \cdot a_{100}}{a_{23}} \right) \Delta T_s - \left( \frac{a_{24} \cdot a_{101}}{a_{23}} \right) \Delta \rho_s$$

or,

$$\Delta W_B = b_{73} \Delta P_B - b_{74} \Delta T_s - b_{75} \Delta \rho_s \quad (1.31)$$

From (18)

$$\Delta W_s = \frac{a_{94}}{a_{93}} \Delta X_v + \frac{a_{95}}{a_{93}} \Delta P_s + \frac{a_{96}}{a_{93}} \Delta T_s \quad (1.32)$$

Substituting  $\Delta P_s$  from (23) into (1.32)

$$\Delta W_s = \frac{a_{94}}{a_{93}} \Delta X_v + \left( \frac{a_{95} \cdot a_{100}}{a_{93}} + \frac{a_{96}}{a_{93}} \right) \Delta T_s + \frac{a_{95} \cdot a_{101}}{a_{93}} \Delta \rho_s$$

or,

$$\Delta W_s = b_{76} \Delta X_v + b_{77} \Delta T_s + b_{78} \Delta \rho_s \quad (1.33)$$

Substituting  $\Delta W_B$  from (1.31) and  $\Delta W_s$  from (1.33) into (1.1) and rearranging :

$$\begin{aligned} \Delta \dot{\rho}_s = & (b_2 \cdot b_{78} - b_1 \cdot b_{75}) \Delta \rho_s + (b_2 \cdot b_{77} - b_1 \cdot b_{74}) \Delta T_s \\ & + (b_1 \cdot b_{73}) \Delta P_B + (b_2 \cdot b_{76}) \Delta X_v \end{aligned}$$

or,

$$\Delta \dot{\rho}_s = c_{11} \Delta \rho_s + c_{12} \Delta T_s + c_{13} \Delta P_B + d_{11} \Delta X_v \quad (A)$$

Substituting  $\Delta W_B$  from (1.31) into (1.4) and rearranging :



$$\dot{\Delta T}_S = b_4 \cdot b_{75} \Delta \rho_S + (b_3 + b_4 \cdot b_{74}) \Delta T_S - b_3 \Delta T_{SW} - b_4 b_{73} \Delta P_B$$

or,

$$\dot{\Delta T}_S = C_{21} \Delta \rho_S + C_{22} \Delta T_S + C_{23} \Delta T_{SW} + C_{24} \Delta P_B \quad (B)$$

Substituting  $\Delta W_B$  from (1.31) into (1.11) and rearranging :

$$\begin{aligned} \dot{\Delta T}_{SW} = & -b_{15} \cdot b_{75} \Delta \rho_S - (b_{15} \cdot b_{74} + b_{16}) \Delta T_S + b_{13} \Delta T_{SW} \\ & + b_{15} \cdot b_{73} \Delta P_B + b_{12} \Delta W_f + b_{14} \Delta W_a \end{aligned}$$

or,

$$\begin{aligned} \dot{\Delta T}_{SW} = & C_{31} \Delta \rho_S + C_{32} \Delta T_S + C_{33} \Delta T_{SW} + C_{38} \Delta P_B \\ & + D_{32} \Delta W_f + D_{33} \Delta W_a \end{aligned} \quad (C)$$

Substituting  $\Delta W_B$  from (1.31) into (1.20) and rearranging :

$$\begin{aligned} \dot{\Delta X} = & b_{42} \cdot b_{7a} \Delta \rho_S + b_{42} \cdot b_{74} \Delta T_S + b_{38} \Delta X - b_{37} \Delta W \\ & + b_{39} \Delta W_W - b_{43} \Delta T_{BW} + (b_{36} - b_{42} \cdot b_{73}) \Delta P_B \\ & + b_{40} \Delta T_W + b_{41} \Delta W_i \end{aligned}$$

or,

$$\begin{aligned} \dot{\Delta X} = & C_{41} \Delta \rho_S + C_{42} \Delta T_S + C_{44} \Delta X + C_{45} \Delta W + C_{46} \Delta W_W \\ & + C_{47} \Delta T_{BW} + C_{48} \Delta P_B + C_{49} \Delta T_W + D_{44} \Delta W_i \end{aligned} \quad (D)$$





Substituting  $\Delta W_B$  from (1.31) into (1.25) and rearranging :

$$\begin{aligned}\Delta \dot{W} = & -b_{66} \cdot b_{73} \Delta P_S - b_{66} \cdot b_{74} \Delta T_S - b_{63} \Delta X + b_{62} \Delta W + b_{60} \Delta W_W \\ & + b_{67} \Delta T_{BW} + (b_{66} \cdot b_{73} + b_{61}) \Delta P_B - b_{64} \Delta T_W + b_{65} \Delta W_i\end{aligned}$$

or,

$$\begin{aligned}\Delta \dot{W} = & C_{51} \Delta P_S + C_{52} \Delta T_S + C_{54} \Delta X + C_{55} \Delta W + C_{56} \Delta W_W \\ & + C_{57} \Delta T_{BW} + C_{58} \Delta P_B + C_{59} \Delta T_W + D_{54} \Delta W_i\end{aligned} \quad (E)$$

Substituting  $\Delta W_B$  from (1.31) into (1.22) and rearranging :

$$\begin{aligned}\Delta \dot{W}_W = & -b_{50} \cdot b_{73} \Delta P_S - b_{50} \cdot b_{74} \Delta T_S - b_{46} \Delta X + b_{45} \Delta W \\ & + b_{47} \Delta W_W + b_{51} \Delta T_{BW} + (b_{50} \cdot b_{73} - b_{44}) \Delta P_B \\ & - b_{48} \Delta T_W + b_{49} \Delta W_i\end{aligned}$$

or,

$$\begin{aligned}\Delta \dot{W}_W = & C_{61} \Delta P_S + C_{62} \Delta T_S + C_{64} \Delta X + C_{65} \Delta W + C_{66} \Delta W_W \\ & + C_{67} \Delta T_{BW} + C_{68} \Delta P_B + C_{69} \Delta T_W + D_{64} \Delta W_i\end{aligned} \quad (F)$$

From (1.29)

$$\Delta \dot{T}_{BW} = -b_{71} \Delta T_{SW} + b_{68} \Delta T_{BW} + b_{72} \Delta P_B + b_{69} \Delta W_f + b_{70} \Delta W_a$$



or,

$$\Delta \dot{T}_{BW} = C_{73} \Delta T_{SW} + C_{77} \Delta T_{BW} + C_{78} \Delta P_B + D_{72} \Delta W_f + D_{73} \Delta W_a \quad (G)$$

Substituting  $\Delta W_B$  from (1.31) into (1.17) and rearranging :

$$\begin{aligned} \Delta \dot{P}_B = & -b_{35} \cdot b_{73} \Delta P_S - b_{35} \cdot b_{74} \Delta T_S + b_{31} \Delta X + b_{30} \Delta W \\ & - b_{32} \Delta W_W + (b_{29} + b_{35} \cdot b_{73}) \Delta P_B + b_{33} \Delta T_W - b_{34} \Delta W_i \end{aligned}$$

or,

$$\begin{aligned} \Delta \dot{P}_B = & C_{81} \Delta P_S + C_{82} \Delta T_S + C_{84} \Delta X + C_{85} \Delta W + C_{86} \Delta W_W \\ & + C_{88} \Delta P_B + C_{89} \Delta T_W + D_{84} \Delta W_i \end{aligned} \quad (H)$$

From (1.15)

$$\Delta \dot{T}_W = b_{25} \Delta X + b_{24} \Delta W + b_{26} \Delta W_W + b_{23} \Delta P_B + b_{27} \Delta T_W + b_{28} \Delta W_i$$

or,

$$\Delta \dot{T}_W = C_{94} \Delta X + C_{95} \Delta W + C_{96} \Delta W_W + C_{98} \Delta P_B + C_{99} \Delta T_W + D_{94} \Delta W_i \quad (I)$$

From (1.13)

$$\Delta \dot{Y} = b_{19} \Delta X + b_{18} \Delta W + b_{20} \Delta W_W - b_{17} \Delta P_B + b_{21} \Delta T_W + b_{22} \Delta W_i$$

or,

$$\Delta \dot{Y} = C_{104} \Delta X + C_{105} \Delta W + C_{106} \Delta W_W + C_{108} \Delta P_B + C_{109} \Delta T_W + D_{104} \Delta W_i \quad (J)$$



The state variable equations from (A) to (J) were rewritten in matrix notation as shown in Figure 4. The state equations were solved for various percent step changes to input variables ( $\Delta X_v$ ,  $\Delta W_f$ ,  $\Delta W_i$ ), using the IBM simulation language CSMP - III as shown in the next section.



$$\begin{bmatrix} \Delta \dot{\rho}_s \\ \Delta \dot{T}_s \\ \Delta \dot{T}_{sw} \\ \Delta \dot{X} \\ \Delta \dot{W} \\ \Delta \dot{W}_w \\ \Delta \dot{T}_{bw} \\ \Delta \dot{P}_B \\ \Delta \dot{T}_w \\ \Delta \dot{Y} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ C_{21} & C_{22} & C_{23} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ C_{31} & C_{32} & C_{33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ C_{41} & C_{42} & 0 & C_{44} & C_{45} & C_{46} & C_{47} & C_{48} & C_{49} & 0 \\ C_{51} & C_{52} & 0 & C_{54} & C_{55} & C_{56} & C_{57} & C_{58} & C_{59} & 0 \\ C_{61} & C_{62} & 0 & C_{64} & C_{65} & C_{66} & C_{67} & C_{68} & C_{69} & 0 \\ 0 & 0 & C_{73} & 0 & 0 & 0 & 0 & C_{77} & C_{78} & 0 \\ C_{81} & C_{82} & 0 & C_{84} & C_{85} & C_{86} & 0 & C_{88} & C_{89} & 0 \\ 0 & 0 & 0 & C_{94} & C_{95} & C_{96} & 0 & C_{98} & C_{99} & 0 \\ 0 & 0 & 0 & C_{104} & C_{105} & C_{106} & 0 & C_{108} & C_{109} & 0 \end{bmatrix} \begin{bmatrix} \Delta \rho_s \\ \Delta T_s \\ \Delta T_{sw} \\ \Delta X \\ \Delta W \\ \Delta W_w \\ \Delta T_{bw} \\ \Delta P_B \\ \Delta T_w \\ \Delta Y \end{bmatrix} + \begin{bmatrix} D_{11} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & D_{32} & D_{33} & 0 & 0 \\ 0 & 0 & 0 & 0 & D_{44} \\ 0 & 0 & 0 & 0 & D_{54} \\ 0 & 0 & 0 & 0 & D_{64} \\ 0 & D_{72} & D_{73} & 0 & 0 \\ 0 & 0 & 0 & 0 & D_{84} \\ 0 & 0 & 0 & 0 & D_{94} \\ 0 & 0 & 0 & 0 & D_{104} \end{bmatrix} \begin{bmatrix} \Delta X_v \\ \Delta W_F \\ \Delta W_A \\ \Delta W_r \end{bmatrix}$$

Figure 4 - STATE VARIABLE MATRIX





### III. COMPUTER RESULTS

From Appendix A , it is seen that the a coefficients are in terms of steady state values. These values were determined from the data in Whalleys thesis (2) which are repeated in Appendix E. A Fortran IV program to find the a,b,C,D coefficients which appear in the CSMP program is shown in Appendix F, and the calculated values are shown in Appendix G. Appendix H is the IBM simulation language CSMP program that was developed. Only input variables were changed for each computer run. For example, results for a 5% step change of  $\Delta x_v$ , a 10% step change of  $\Delta W_i$  and a 10% step change of  $\Delta W_i$  were calculated. The output response curves were plotted using the NPS CALCOMP PLOTTER.



#### A. TRANSIENT RESPONSES FOLLOWING A STEP CHANGE IN THROTTLE SETTING OF 5%

At a particular steady state operating condition the throttle valve is suddenly opened while still keeping the previous air and fuel flow rates and feedwater flow rate constant. Because of bubble formation in the steam drum liquid, the steam drum water level (FIGURE 9) swells for about 40 seconds and then shows a steady falling in level at higher flow rates of steam. The steam drum pressure (FIGURE 8) immediately begins to fall. Following these sudden increases, the steam mass-flow rates from the steam drum and the superheater (FIGURES 10 and 11) show slight declines because of the effect of the decrease in steam drum pressure. The superheater outlet pressure (FIGURE 10) acts in a similar fashion to the steam drum pressure, that is, it shows a steady decline to a new steady state condition but it is slightly lower than the steam drum pressure because of system pressure drops. The superheated steam temperature and superheater wall temperature (FIGURES 5 and 6) both decline, although the latter temperature is slightly lower. The riser tube wall temperature (FIGURE 8) and steam drum liquid temperature (FIGURE 9) also decrease due to the throttle change. The superheated steam density (FIGURE 5) shows a rising characteristic to counteract the loss in superheater pressure and temperature and the gain in steam mass-flow rate from the steam drum (FIGURE 11). The riser and downcomer flow rates (FIGURE 7) are reduced to a lower value but steam quality (FIGURE 6) is increased to a higher value than previously to preserve the energy balance.



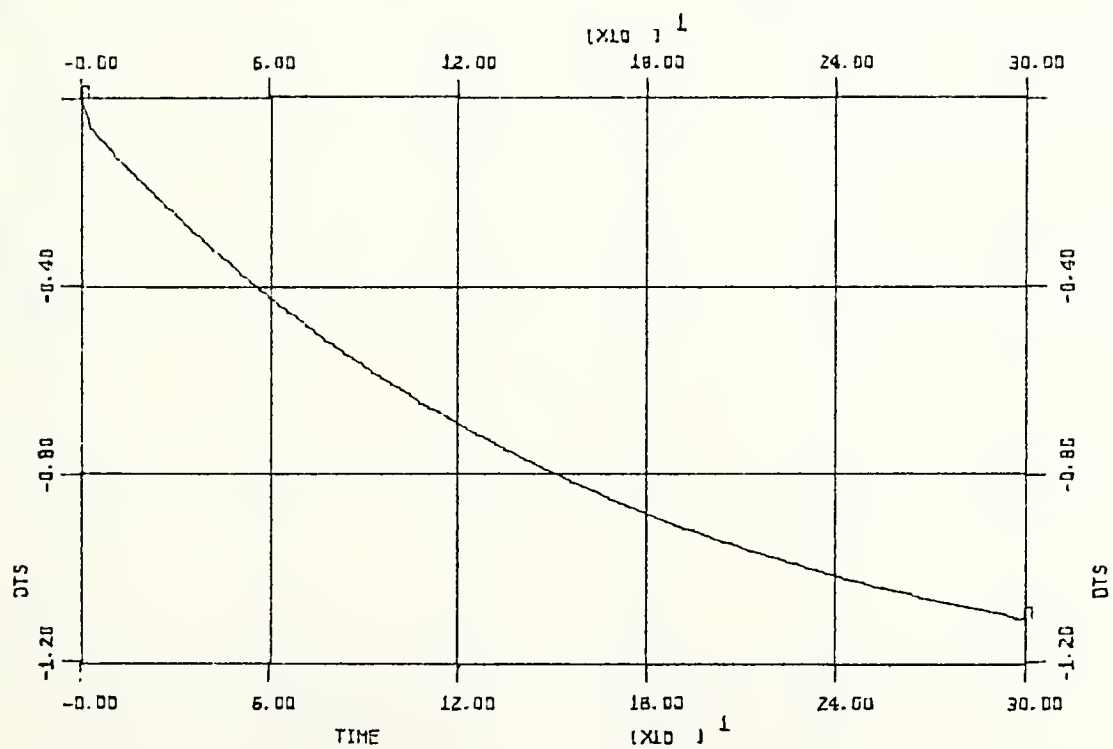
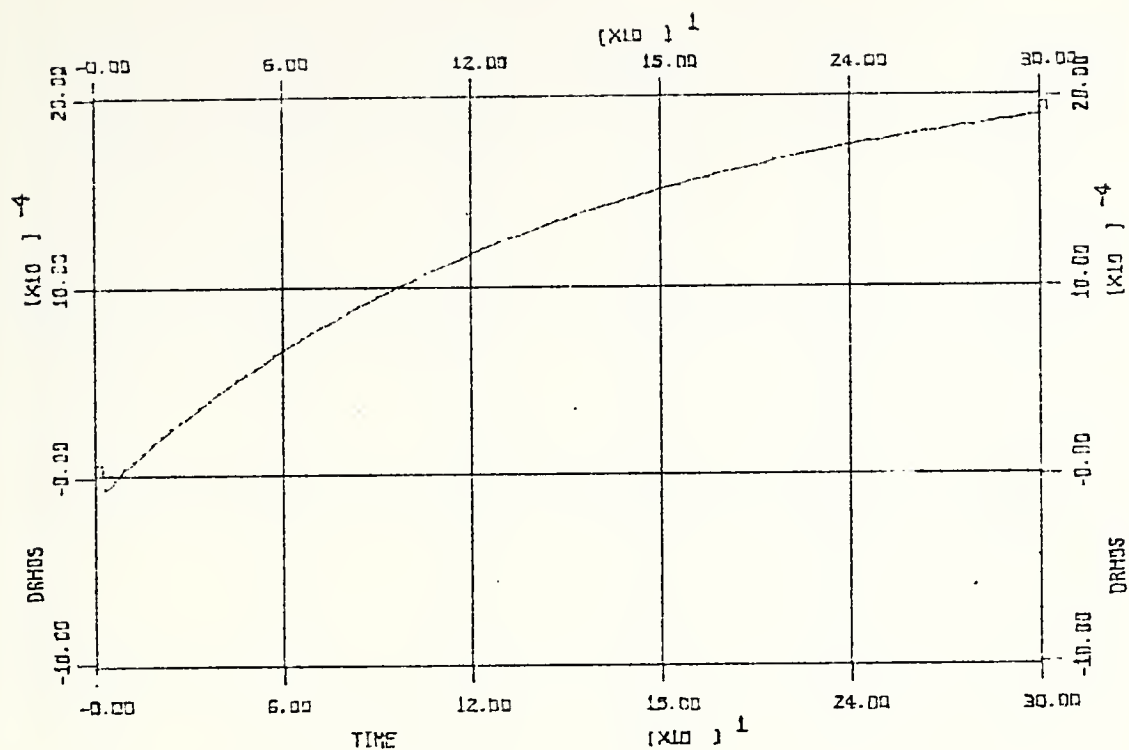


Figure 5 - SUPERHEATED STEAM DENSITY (  $\Delta \rho_s$  ) VS TIME AND  
SUPERHEATED STEAM TEMPERATURE (  $\Delta T_s$  ) VS TIME



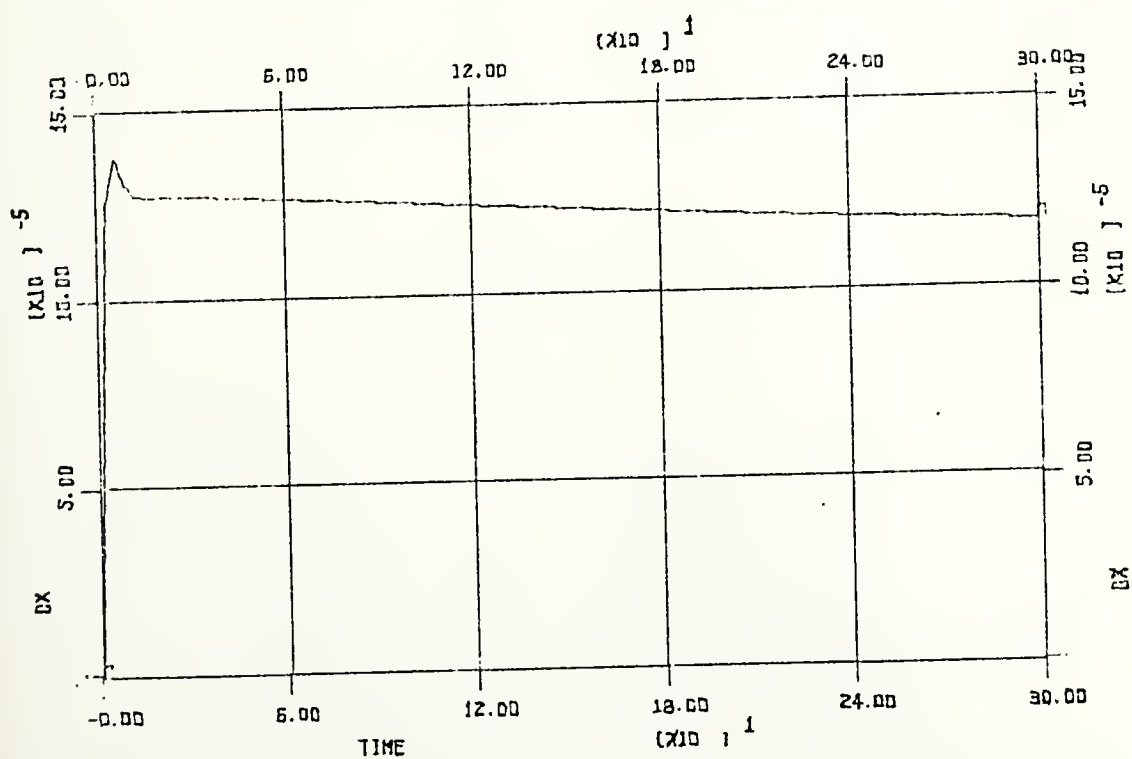
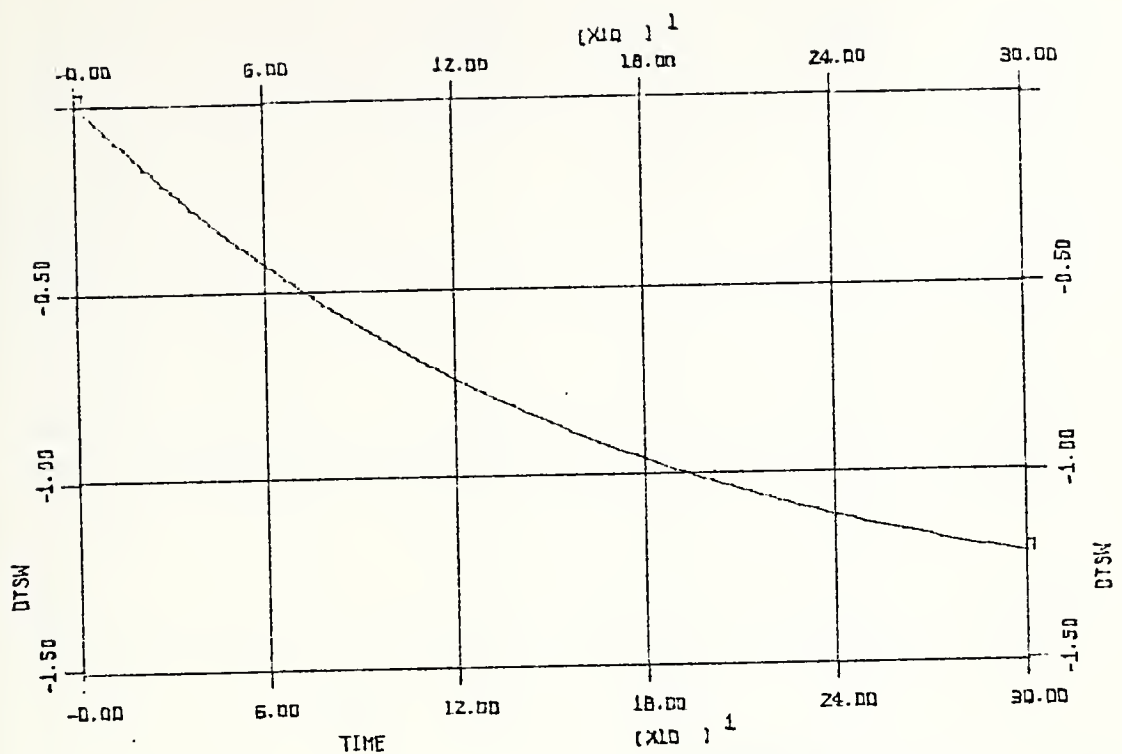


Figure 6 - SUPERHEATER WALL TEMPERATURE (  $\Delta T_{sw}$  ) VS TIME  
AND QUALITY OF MIXTURE LEAVING RISER (  $\Delta x$  ) VS TIME





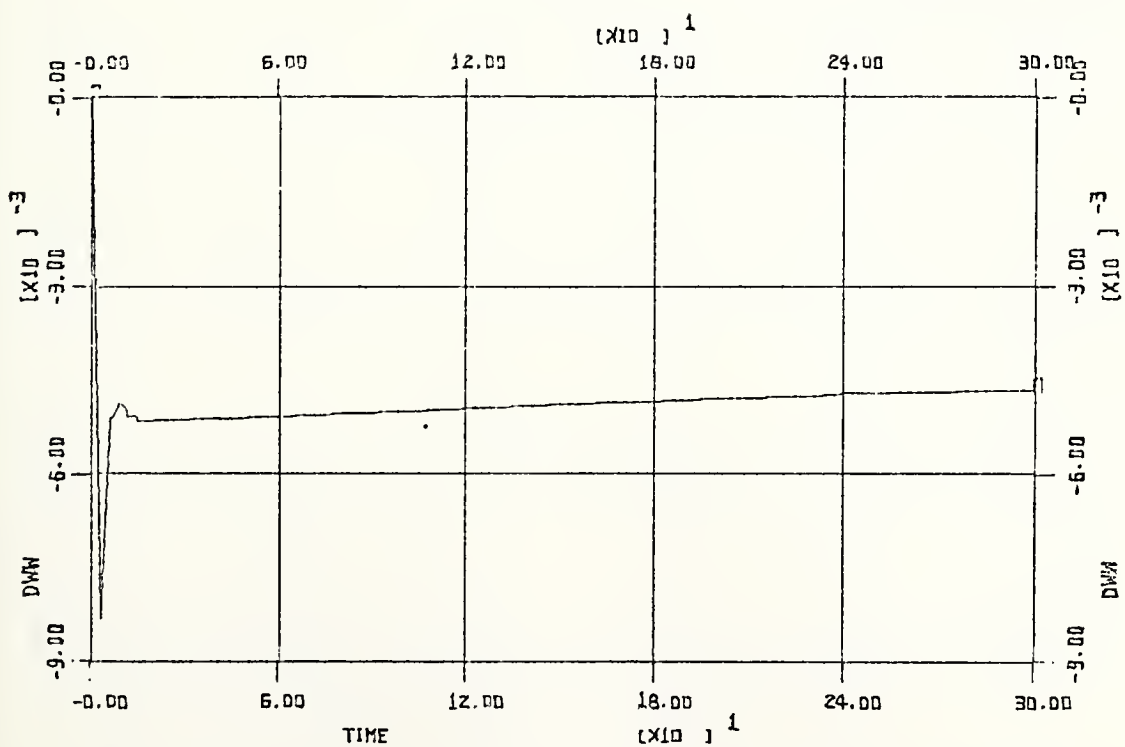
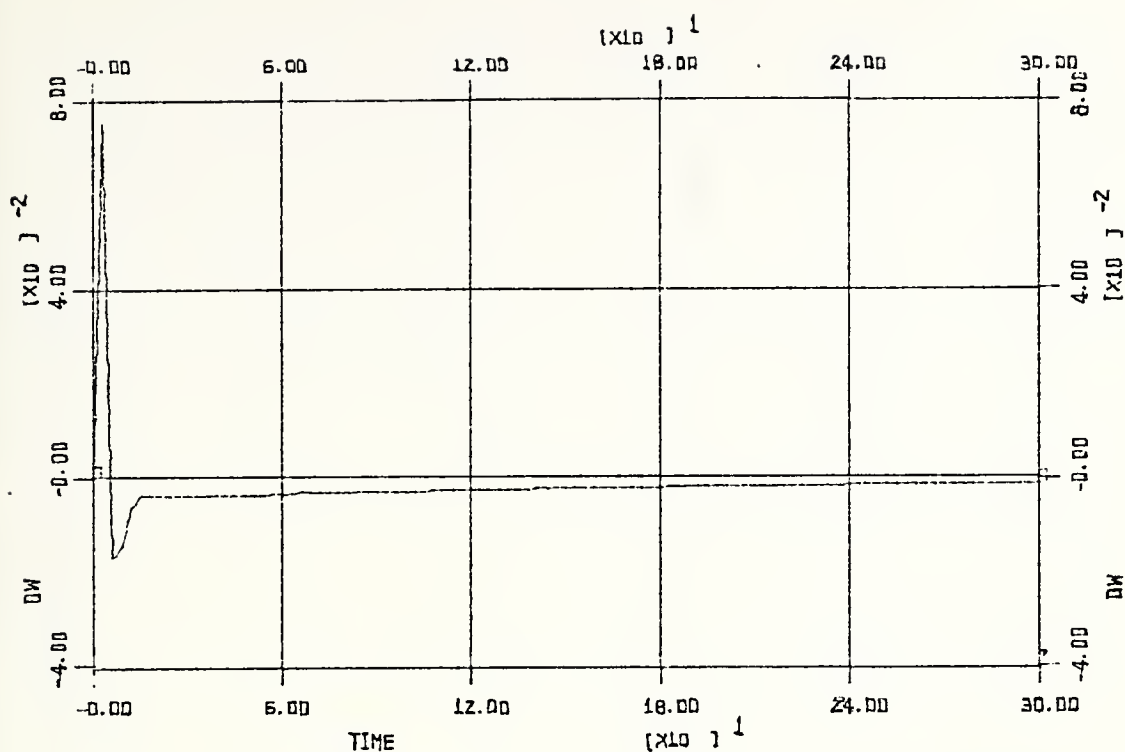


Figure 7 - RISER MIXTURE FLOW RATE (  $\Delta W$  ) VS TIME AND  
DOWNCOMER LIQUID FLOW RATE (  $\Delta W_w$  ) VS TIME



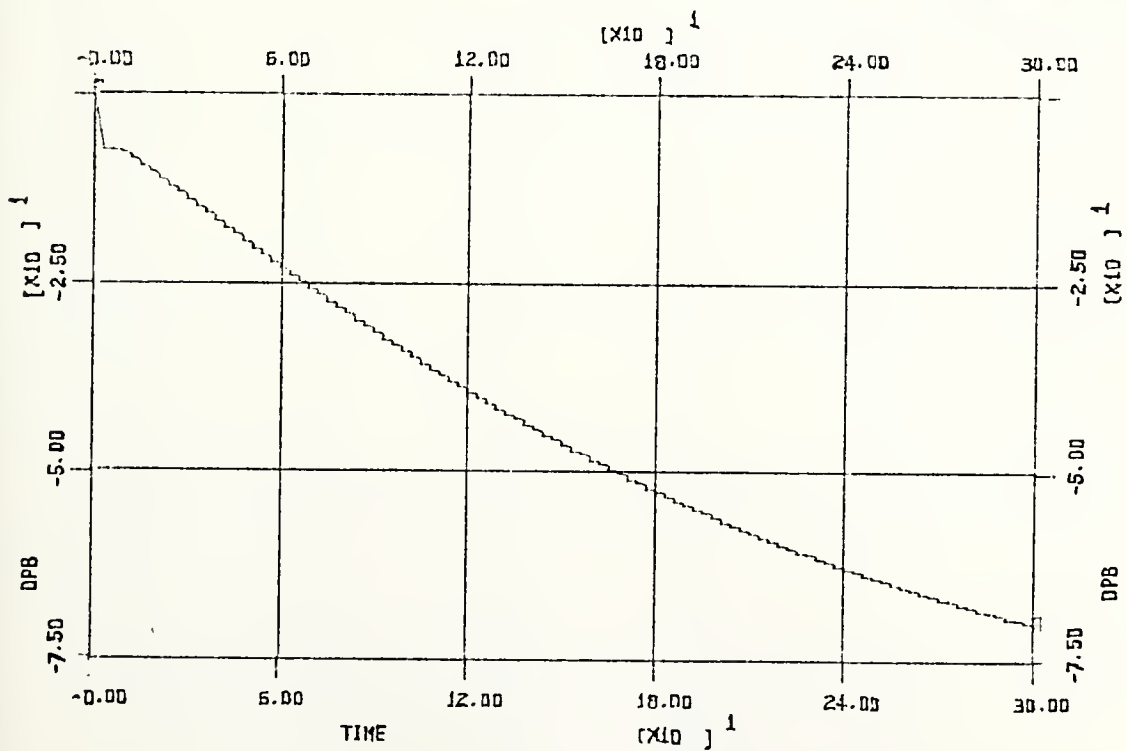
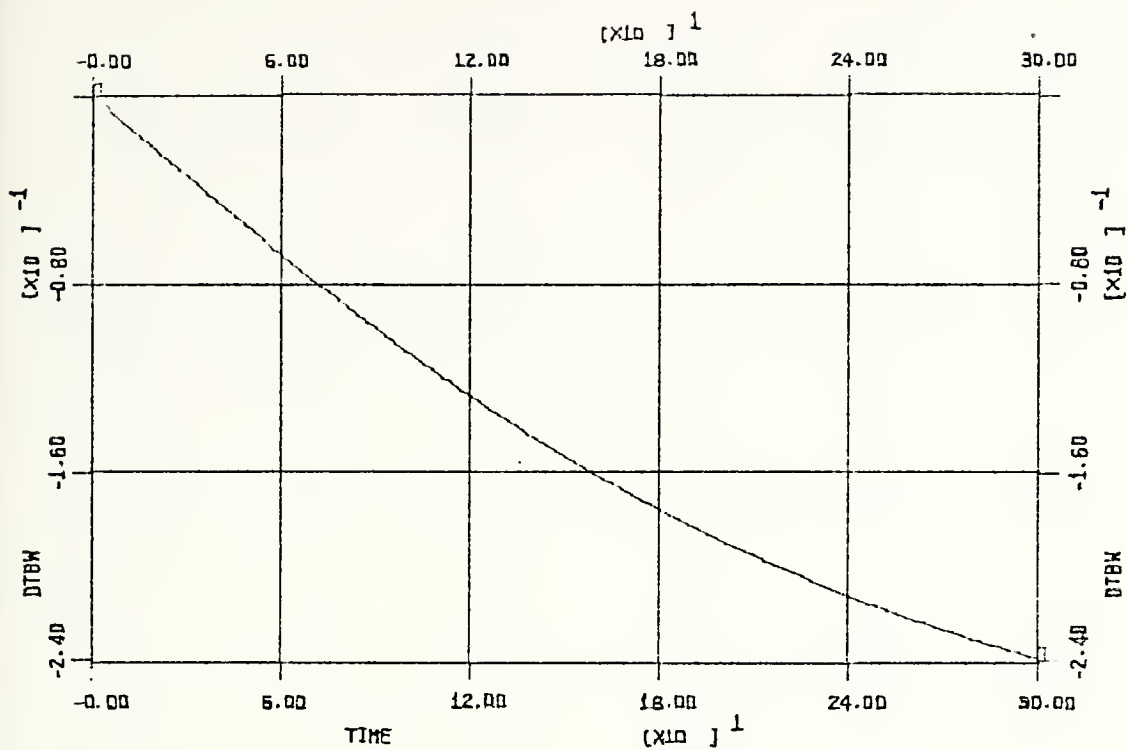


Figure 8 - RISER TUBE WALL TEMPERATURE (  $\Delta T_{BW}$  ) VS TIME AND  
STEAM DRUM PRESSURE (  $\Delta P_B$  ) VS TIME



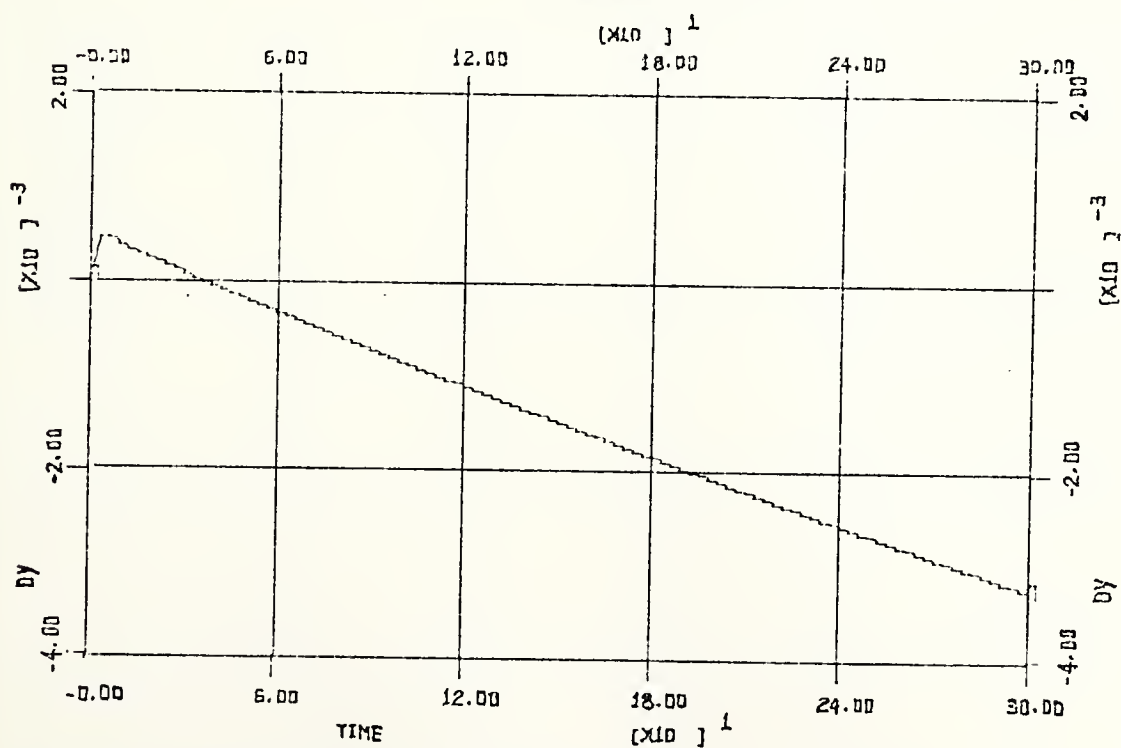
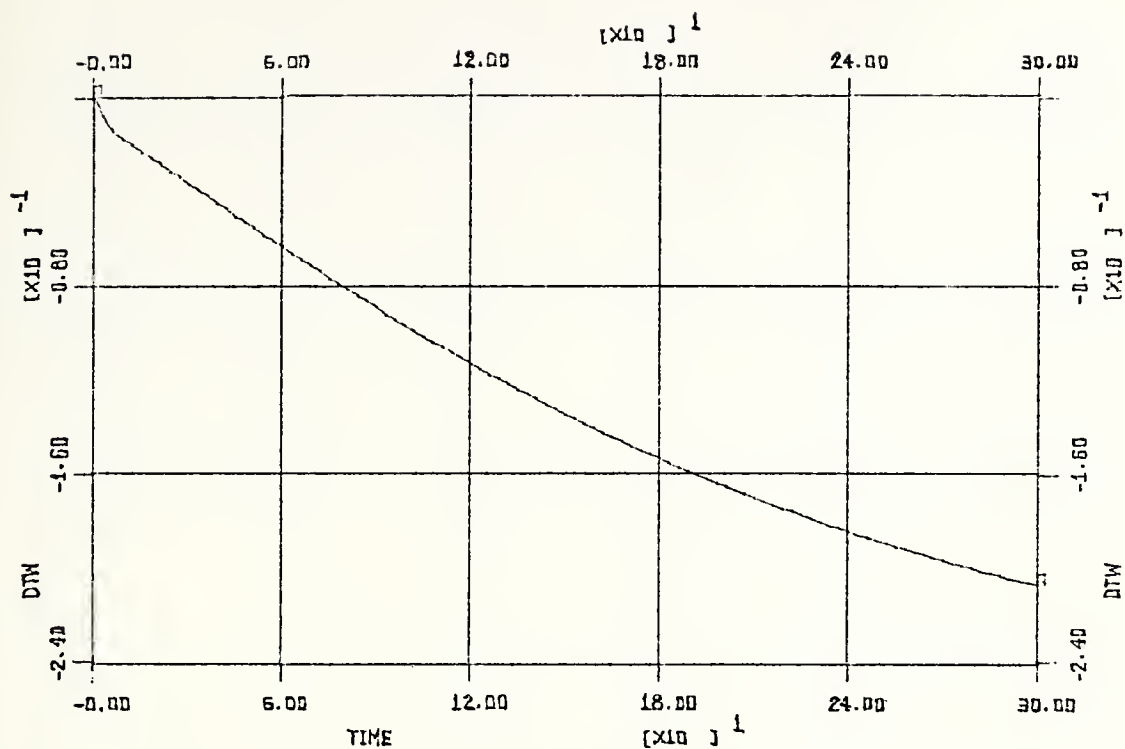


Figure 9 - STEAM DRUM LIQUID TEMPERATURE (  $\Delta T_W$  ) VS TIME  
AND STEAM DRUM WATER LEVEL (  $\Delta y$  ) VS TIME



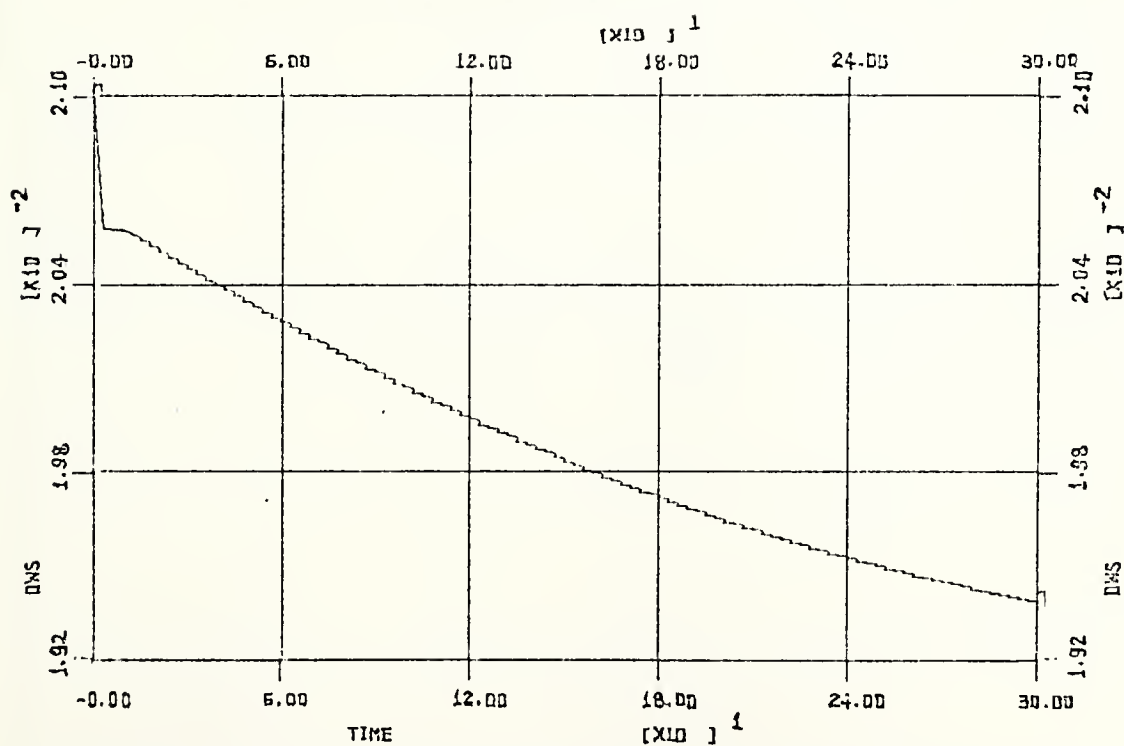
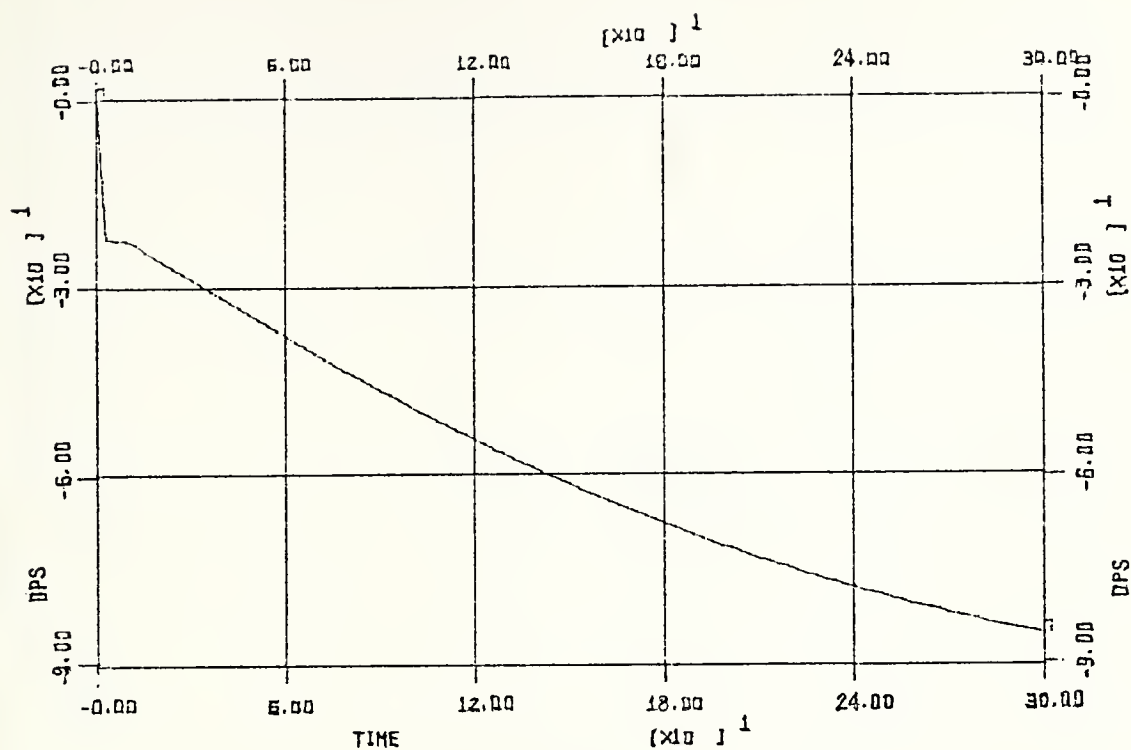


Figure 10 - SUPERHEATER OUTLET PRESSURE (  $\Delta P_s$  ) VS TIME AND STEAM FLOW RATE AT SUPERHEATER OUTLET (  $\Delta W_s$  ) VS TIME





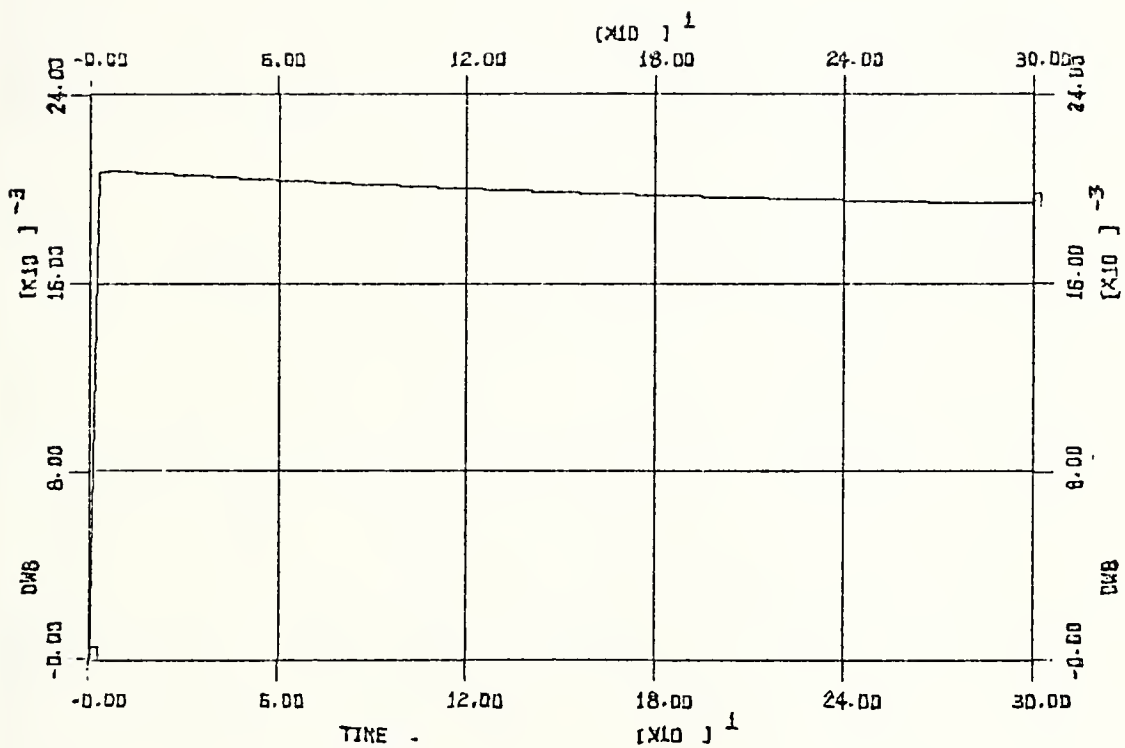


Figure 11 - STEAM MASS-FLOW RATE FROM STEAM DRUM TO  
SUPERHEATER (  $\Delta W_8$  ) VS TIME



## B. TRANSIENT RESPONSES FOLLOWING A STEP CHANGE OF 10% IN FUEL FLOW RATE.

The effects due to a change in the fuel flow rate are less than those due to a change in the throttle valve setting. However, the system requires a longer period of time to reach a new steady state condition. Because the combustion rate is increased, the evaporation rate is also increased making both the steam mass-flow rate at the superheater (FIGURE 17) and the steam mass-flow rate from the steam drum (FIGURE 18) increase in a similar manner. Initially, density in the risers falls quickly in response to the increased firing rate which causes a rise in superheated steam density (FIGURE 12). An increase in steam flow from the steam drum without an increase in the feedwater flow rate results in a drop in steam drum water level (FIGURE 16). The steam drum pressure (FIGURE 15) and superheater outlet pressure (FIGURE 17) rise monotonically as time increases. Also superheated steam temperature (FIGURE 12), superheater wall temperature (FIGURE 13), riser tube wall temperature (FIGURE 15) and steam drum liquid temperature (FIGURE 16) all increase with increasing fuel flow rate. The abrupt fall in riser mixture flow rate (FIGURE 14) is probably due to the sudden drop in the level of the initial density change in the riser.



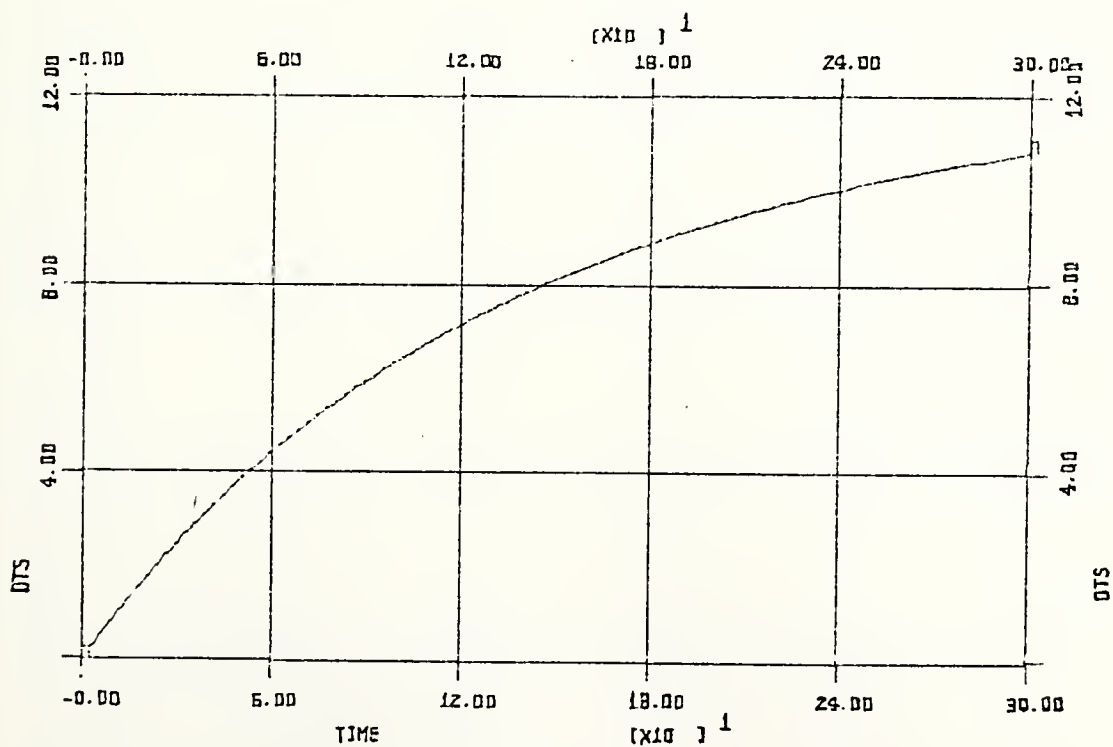
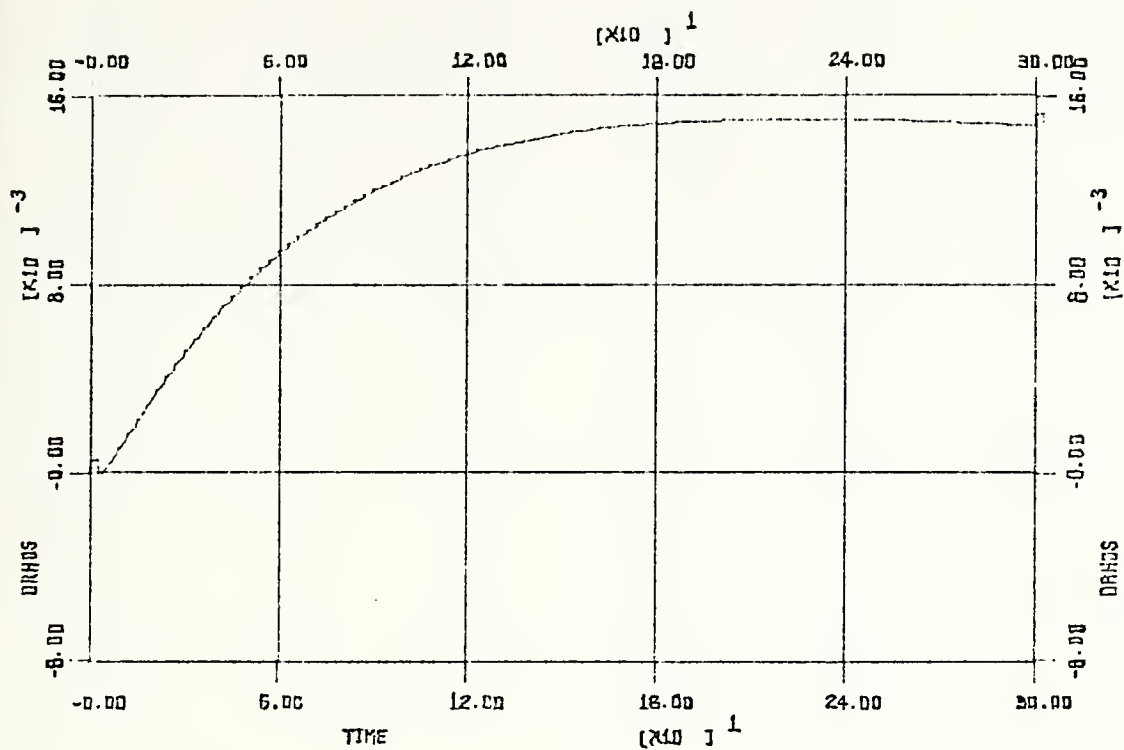


Figure 12 - SUPERHEATED STEAM DENSITY (  $\Delta \rho_s$  ) VS TIME AND SUPERHEATED STEAM TEMPERATURE (  $\Delta T_s$  ) VS TIME



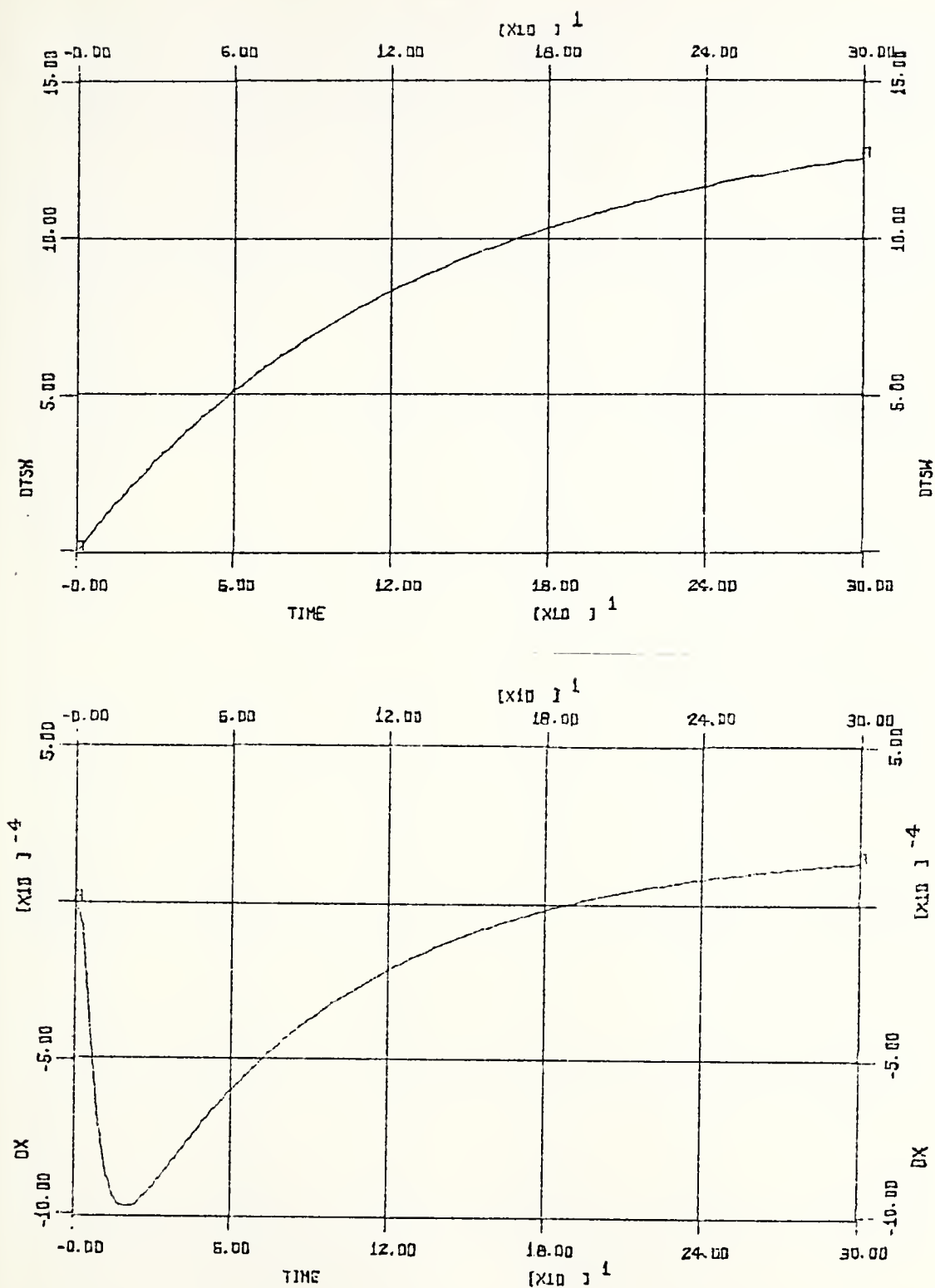


Figure 13 - SUPERHEATER WALL TEMPERATURE (  $\Delta T_{SW}$  ) VS TIME  
 AND QUALITY OF MIXTURE LEAVING RISER (  $\Delta X$  ) VS TIME





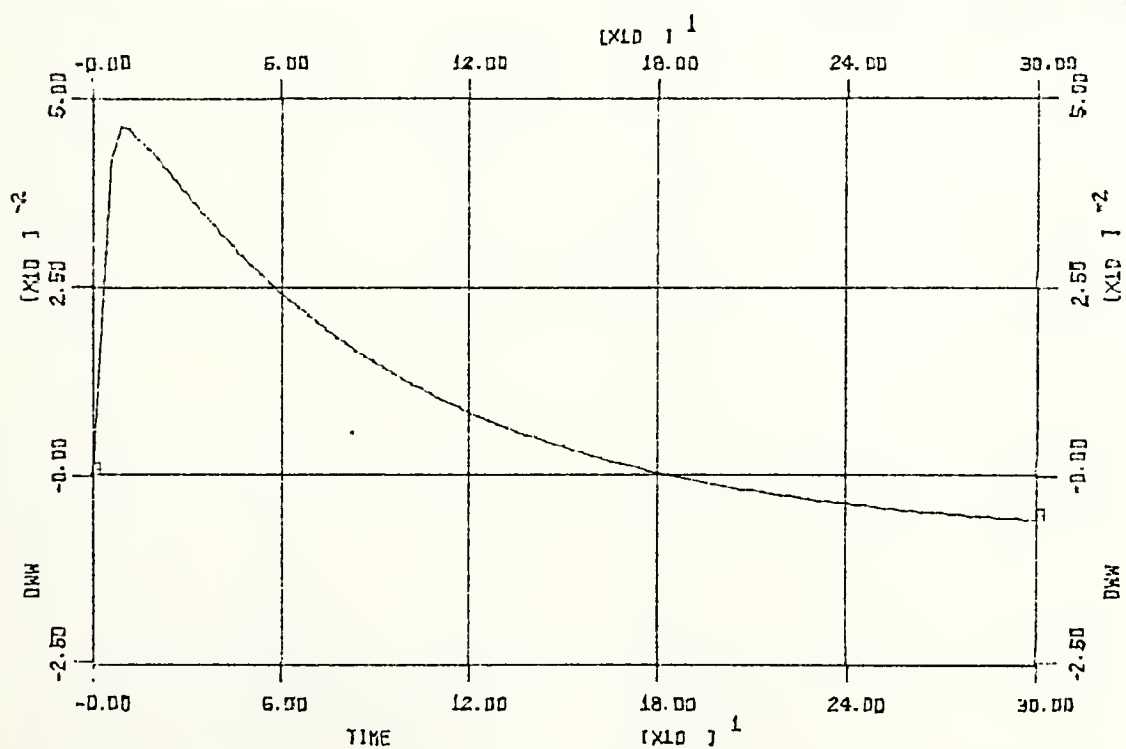
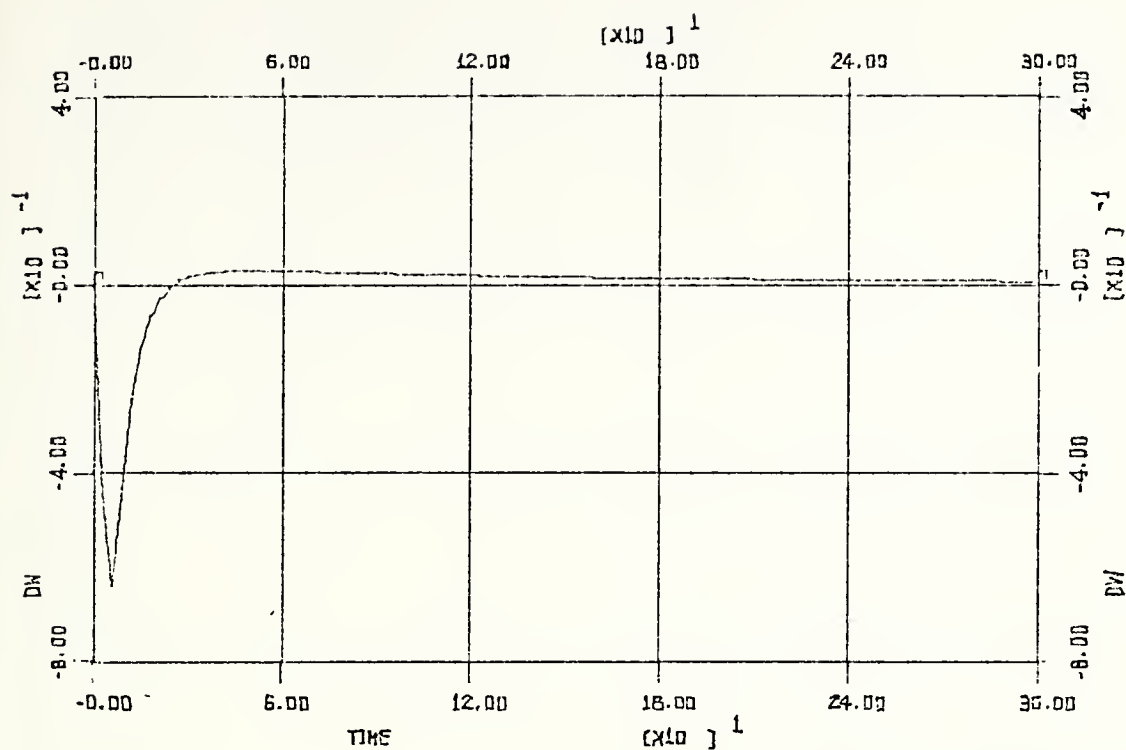


Figure 14 - RISER MIXTURE FLOW RATE (  $\Delta W$  ) VS TIME AND  
DOWNCOMER LIQUID FLOW RATE (  $\Delta W_w$  ) VS TIME



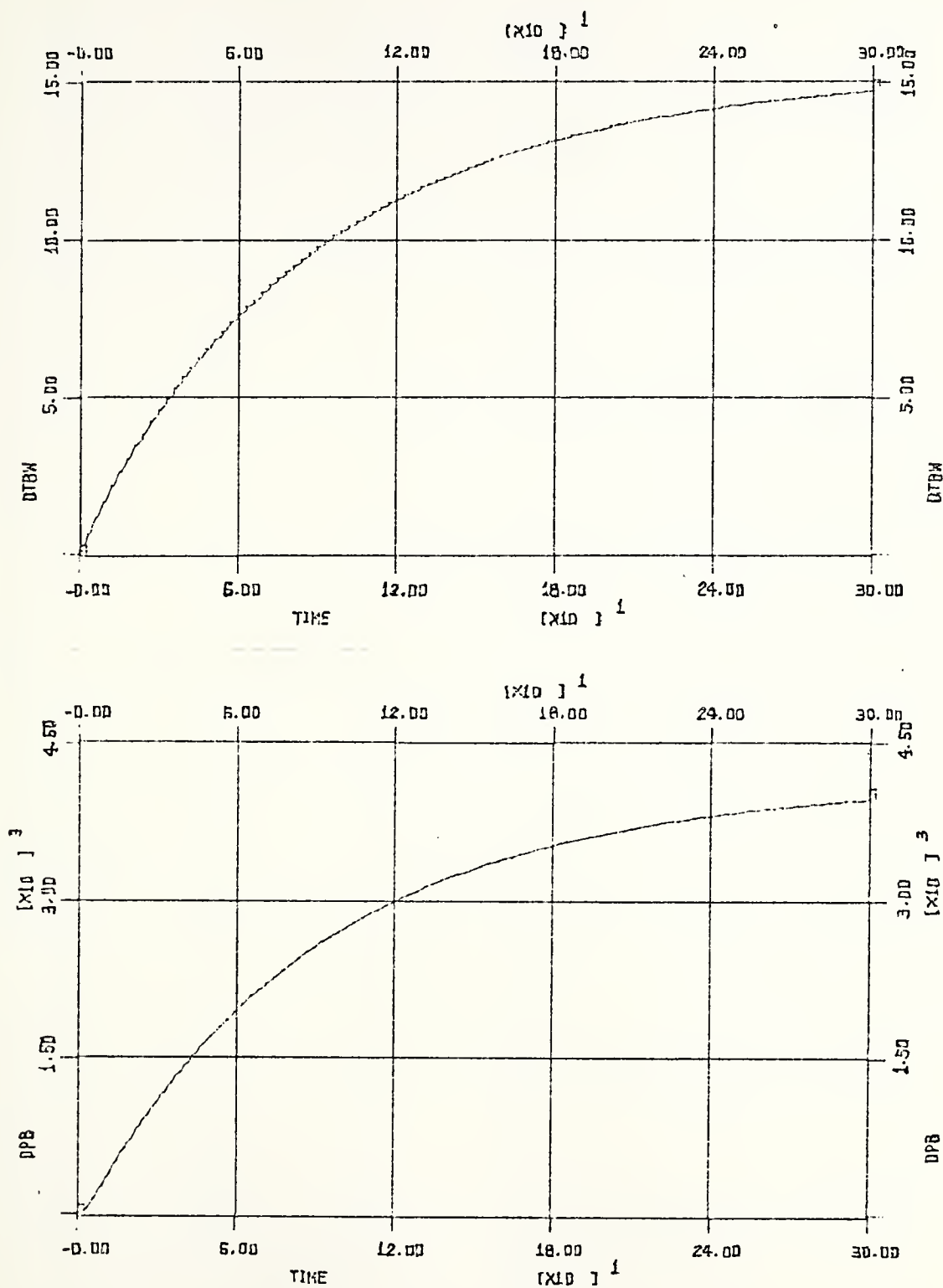


Figure 15 - RISER TUBE WALL TEMPERATURE (  $\Delta T_{BW}$  ) VS TIME  
AND STEAM DRUM PRESSURE (  $\Delta P_B$  ) VS TIME



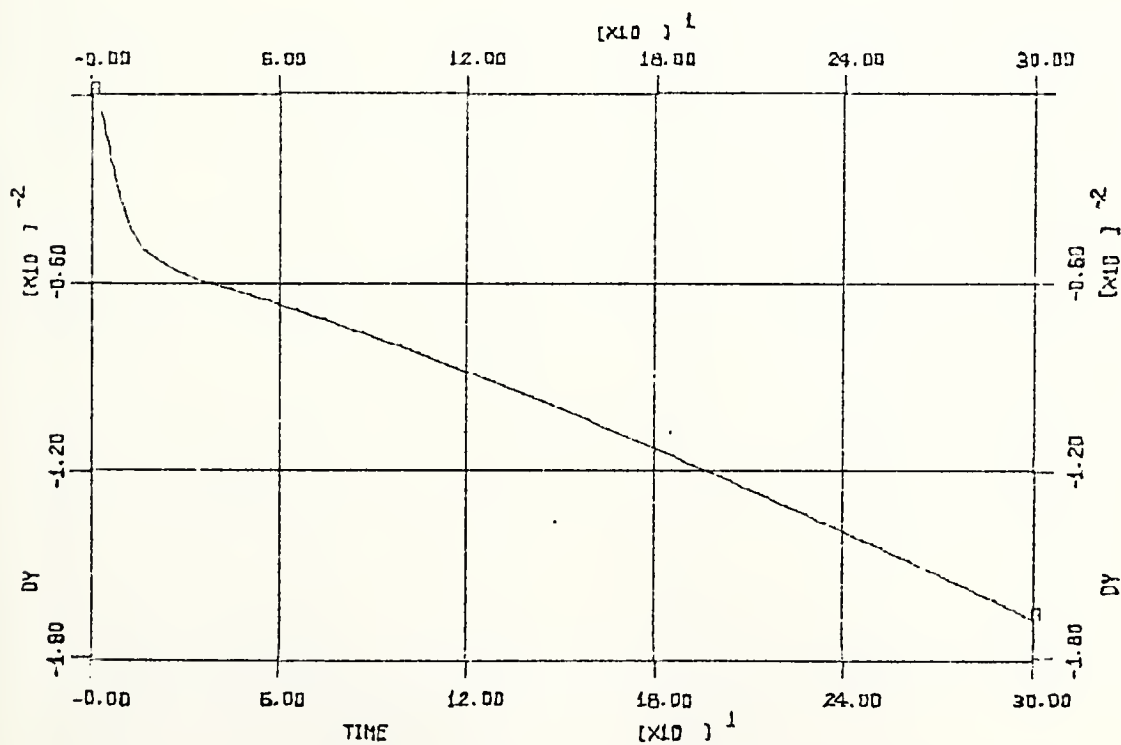
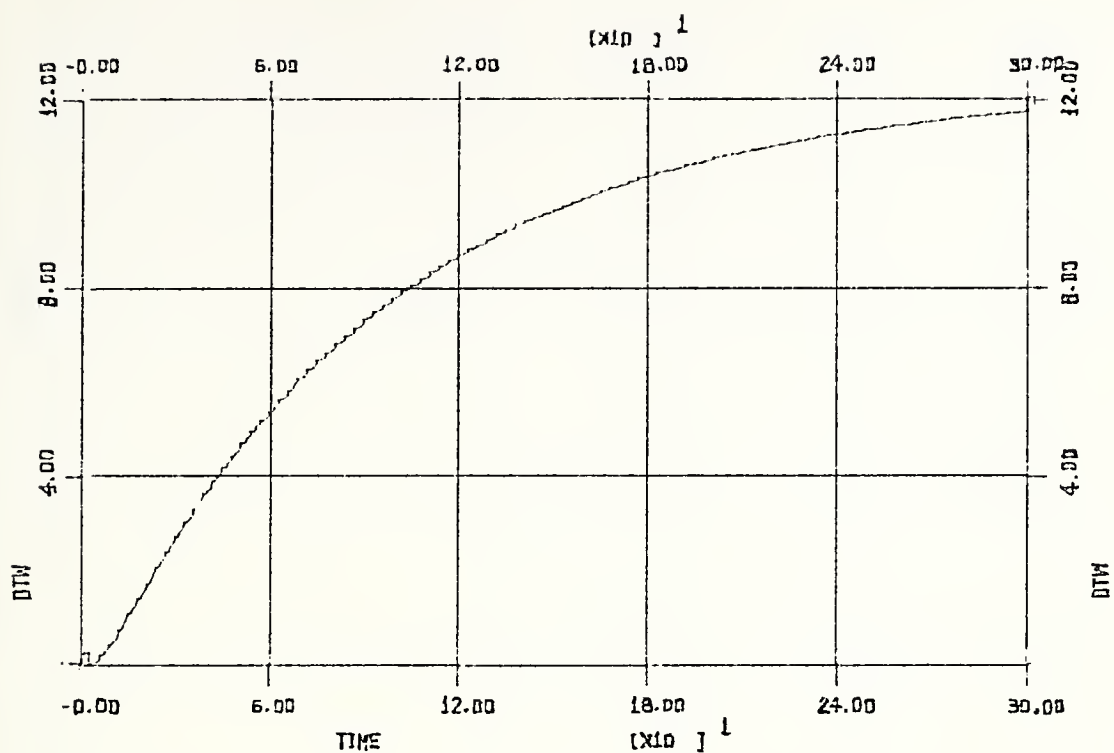


Figure 16 - STEAM DRUM LIQUID TEMPERATURE (  $\Delta T_w$  ) VS TIME  
AND STEAM DRUM WATER LEVEL (  $\Delta y$  ) VS TIME



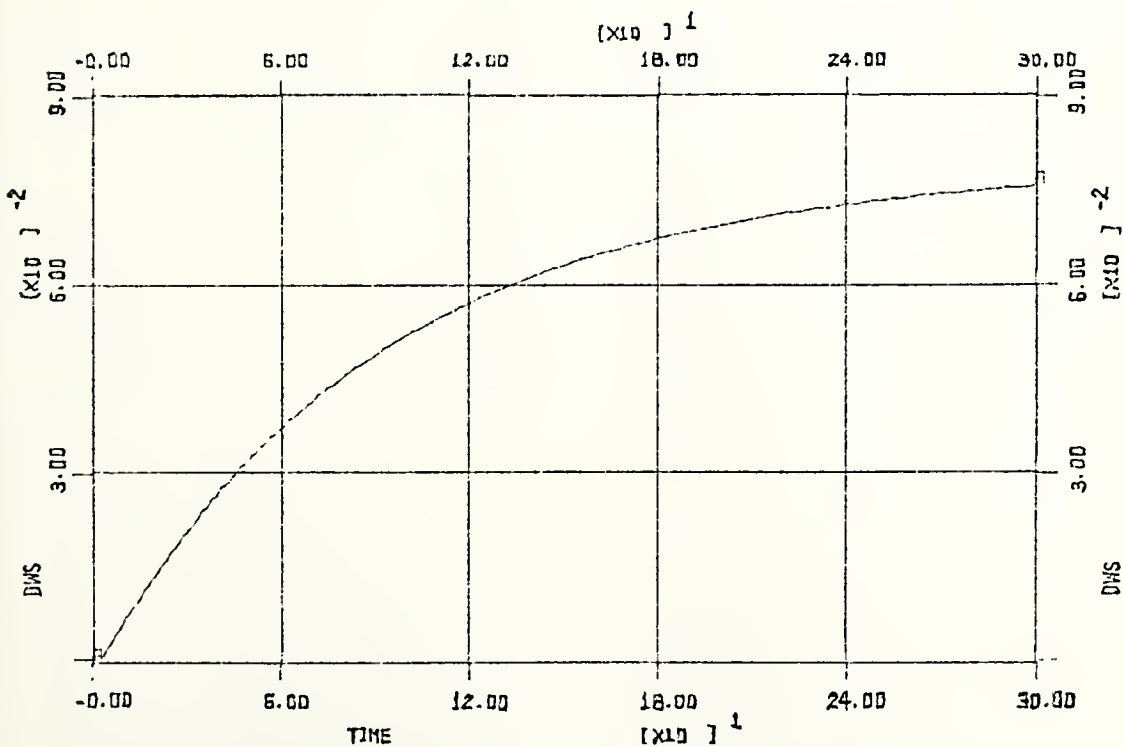
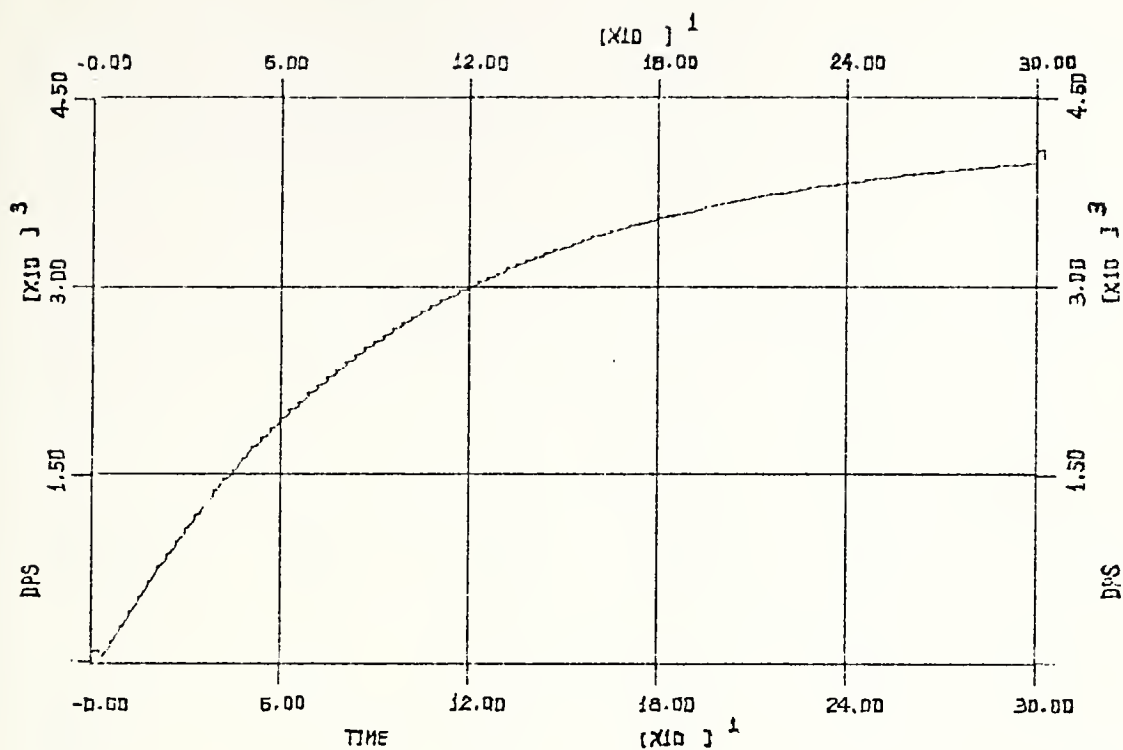


Figure 17 - SUPERHEATER OUTLET PRESSURE (  $\Delta P_s$  ) VS TIME  
AND STEAM FLOW RATE AT SUPERHEATER OUTLET (  $\Delta W_s$  ) VS TIME





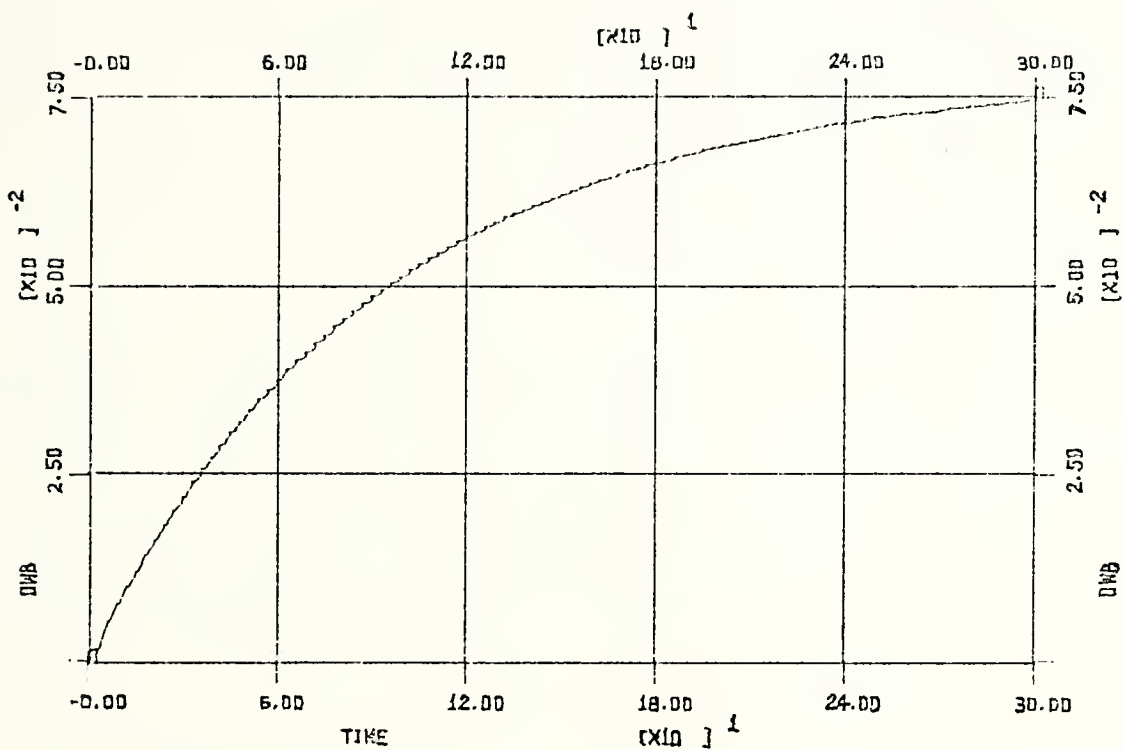


Figure 18 - STEAM MASS-FLOW RATE FROM STEAM DRUM TO SUPERHEATER (  $\Delta W_B$  ) VS TIME



### C. TRANSIENT RESPONSES FOLLOWING A STEP CHANGE OF 10% IN FEEDWATER FLOW-RATE

The main effect of increasing feedwater flow rate while maintaining throttle setting and air-fuel flow rate constant is an increase in the steam drum water level (FIGURE23). Because feedwater is added into the steam drum, steam drum pressure (FIGURE22) suffers a small overshoot followed by a decline. As a result, a drop in both steam mass-flow rate from the steam drum (FIGURE25) and steam mass-flow rate at the superheater outlet (FIGURE24) occurs. Flow around the riser-downcomer loop (FIGURE21) is only slightly affected. Since the firing rate is unaltered, the riser tube wall temperature (FIGURE 22) and the steam drum liquid temperature (FIGURE23) decline. The superheated steam temperature (FIGURE19) and the superheater wall temperature (FIGURE20) rise because of a drop in steam mass-flow rate from the steam drum. The quality of steam (FIGURE20) shows a general lowering. The superheated steam density (FIGURE19) shows a decline which is similar in form to the drop in the superheater outlet pressure.



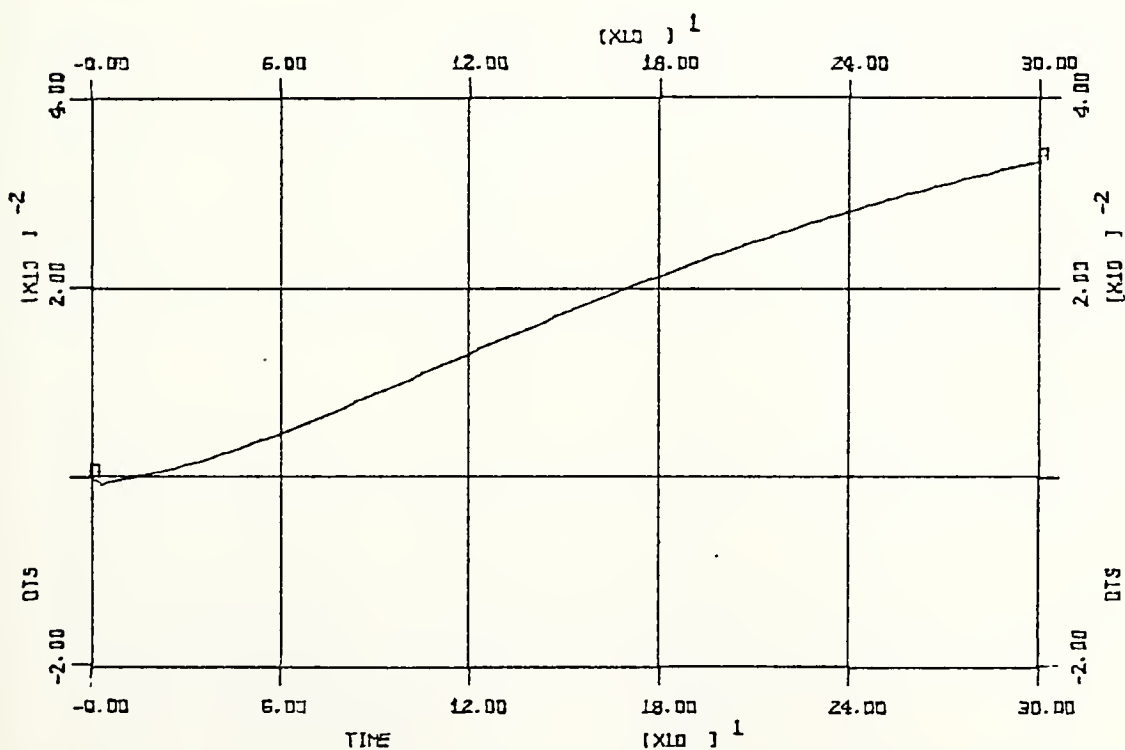
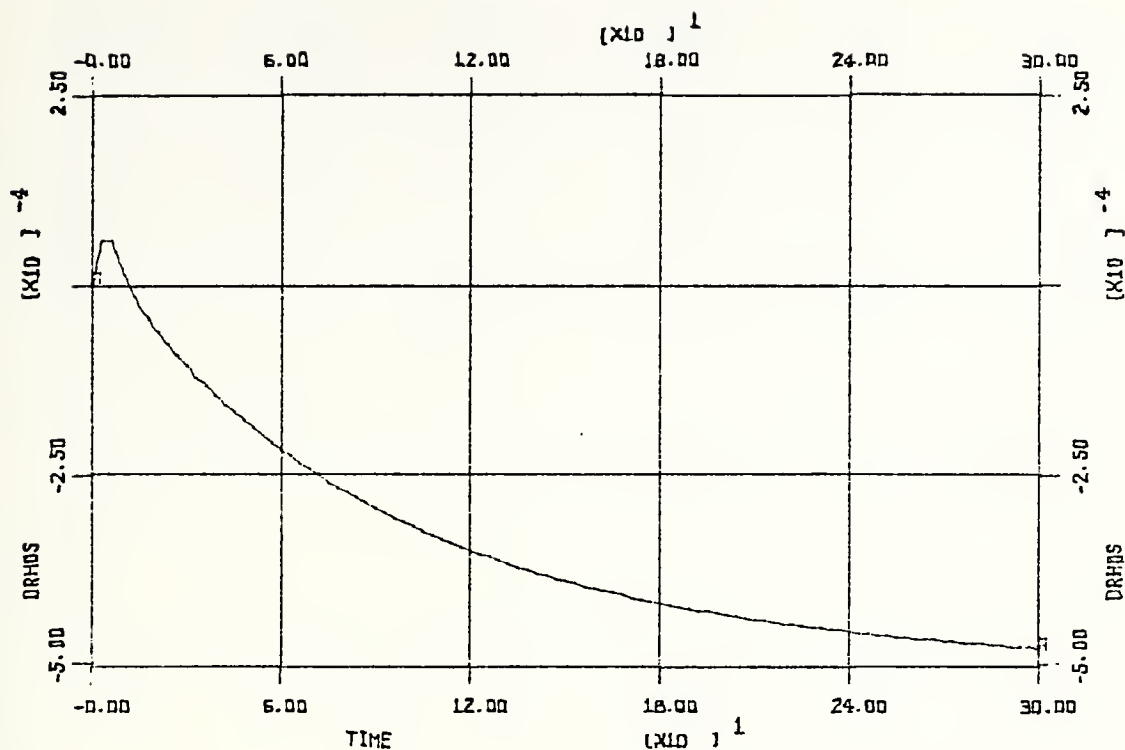


Figure 19 - SUPERHEATED STEAM DENSITY (  $\Delta \rho_s$  ) VS TIME AND  
SUPERHEATED STEAM TEMPERATURE (  $\Delta T_s$  ) VS TIME



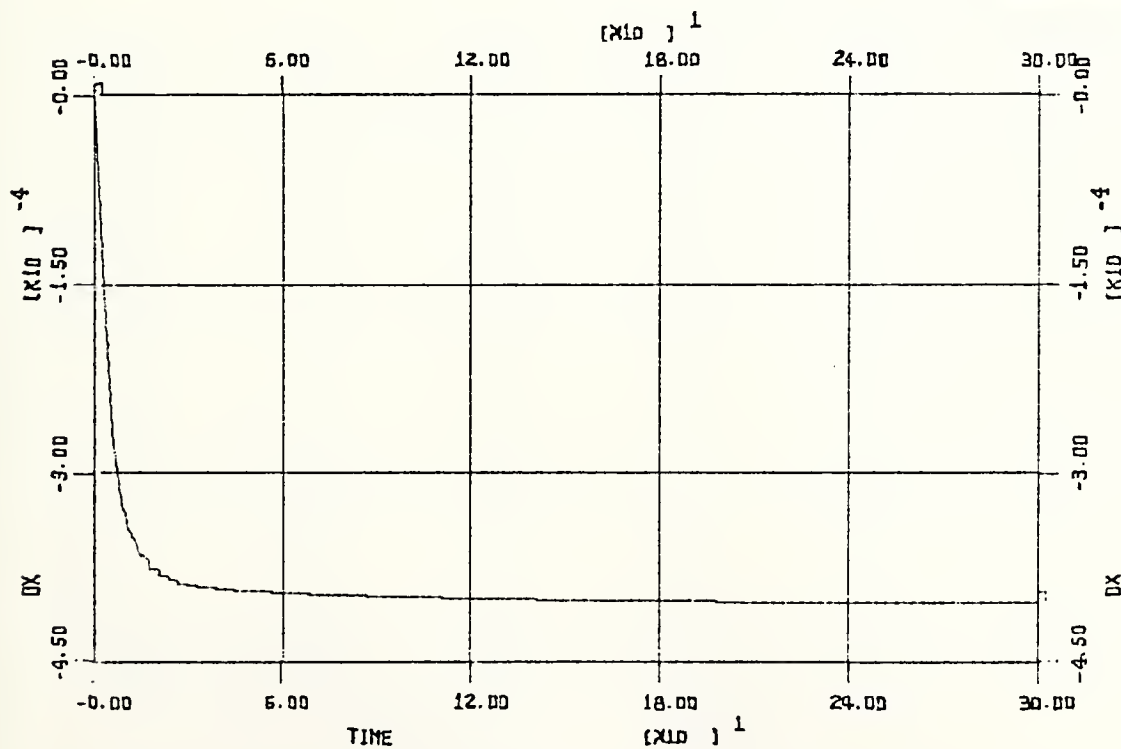
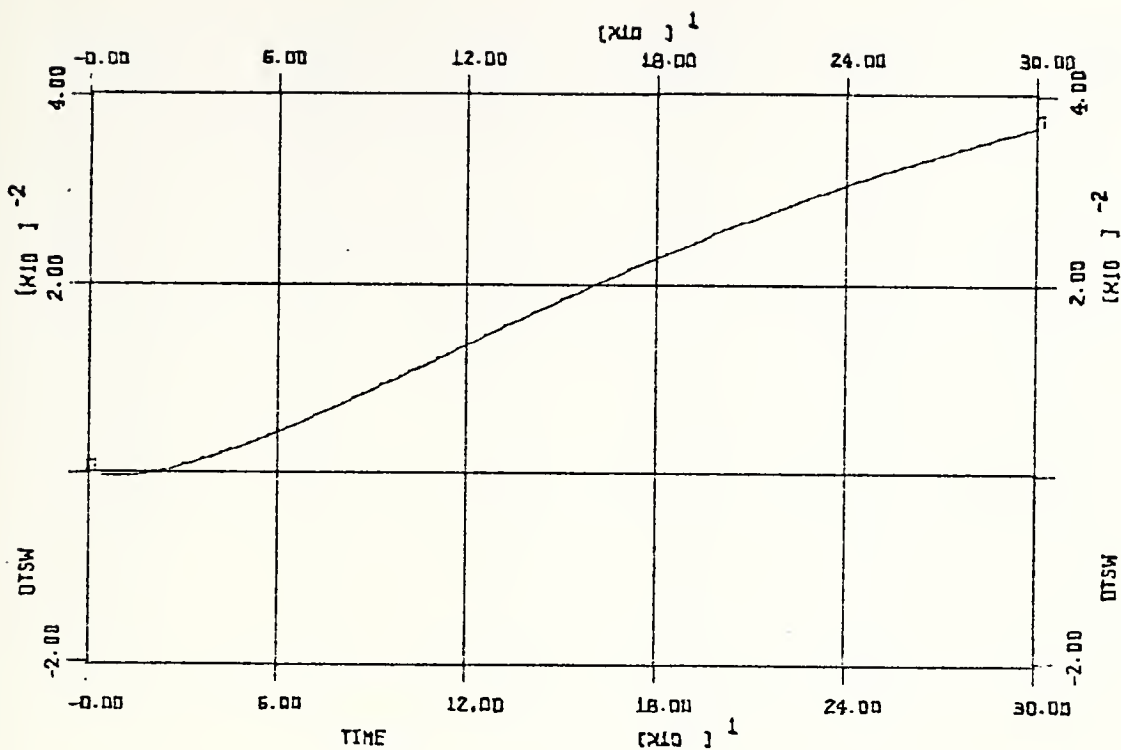


Figure 20 - SUPERHEATER WALL TEMPERATURE ( $\Delta T_{sw}$ ) VS TIME  
AND QUALITY OF MIXTURE LEAVING RISER ( $\Delta X$ ) VS TIME





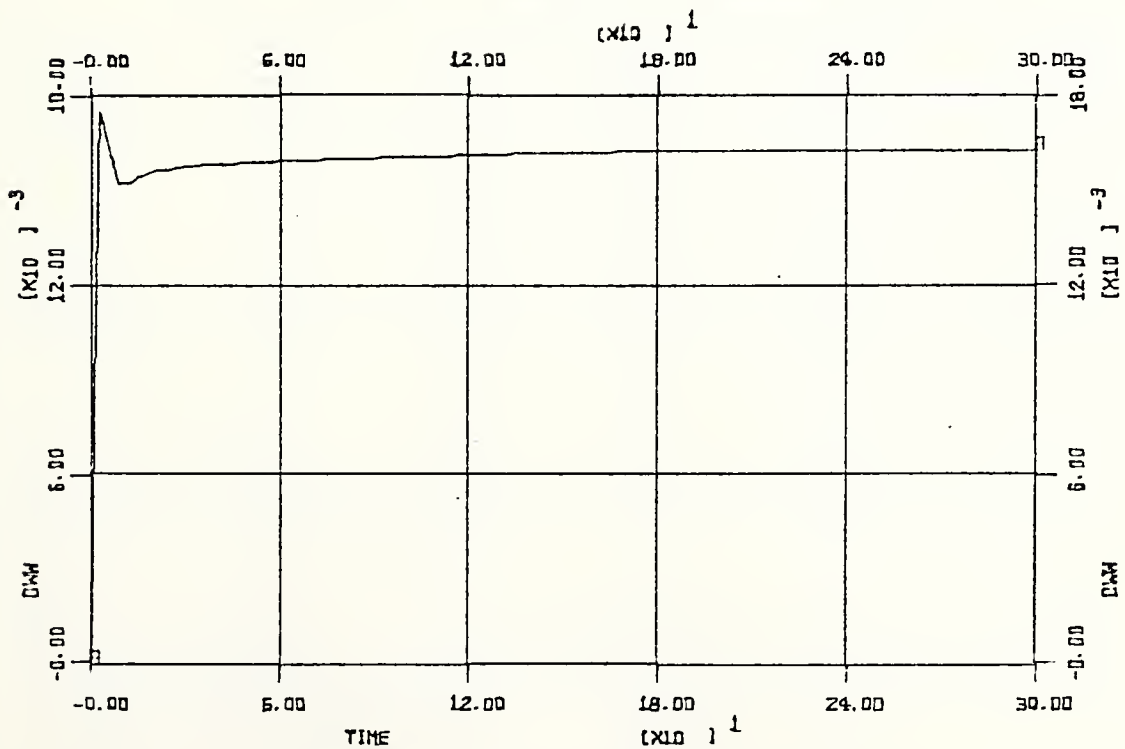
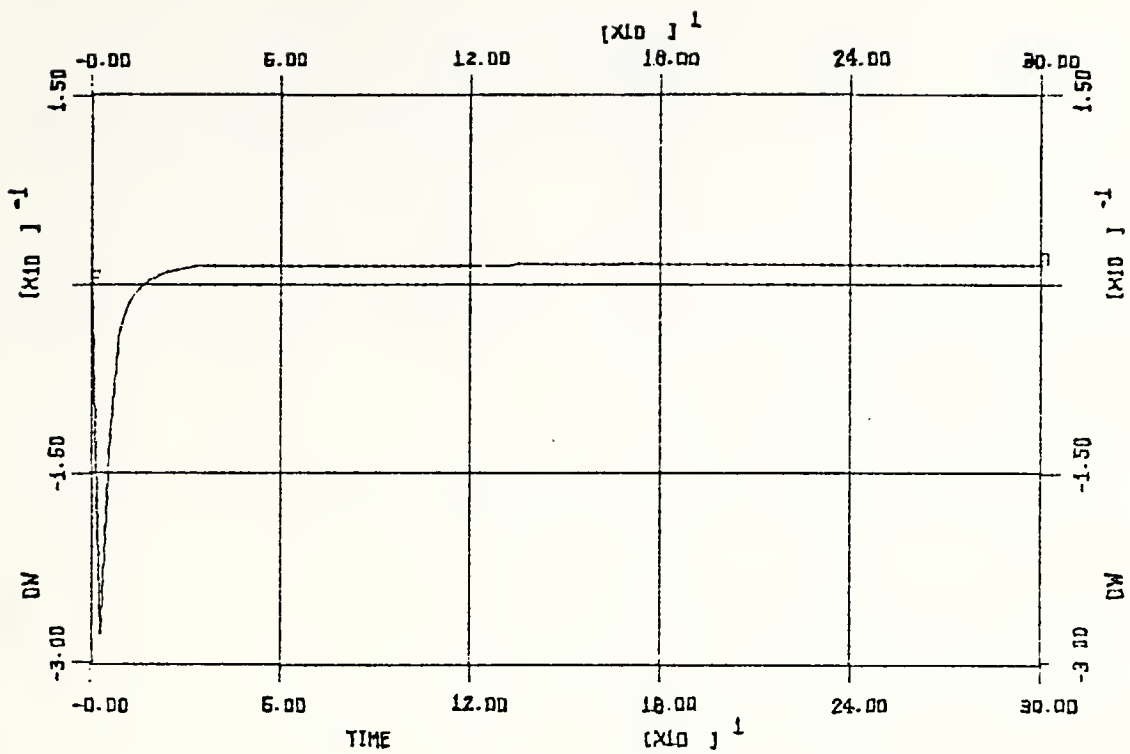


Figure 21 - RISER MIXTURE FLOW RATE (  $\Delta W$  ) VS TIME AND  
DOWNCOMER LIQUID FLOW RATE (  $\Delta W_w$  ) VS TIME



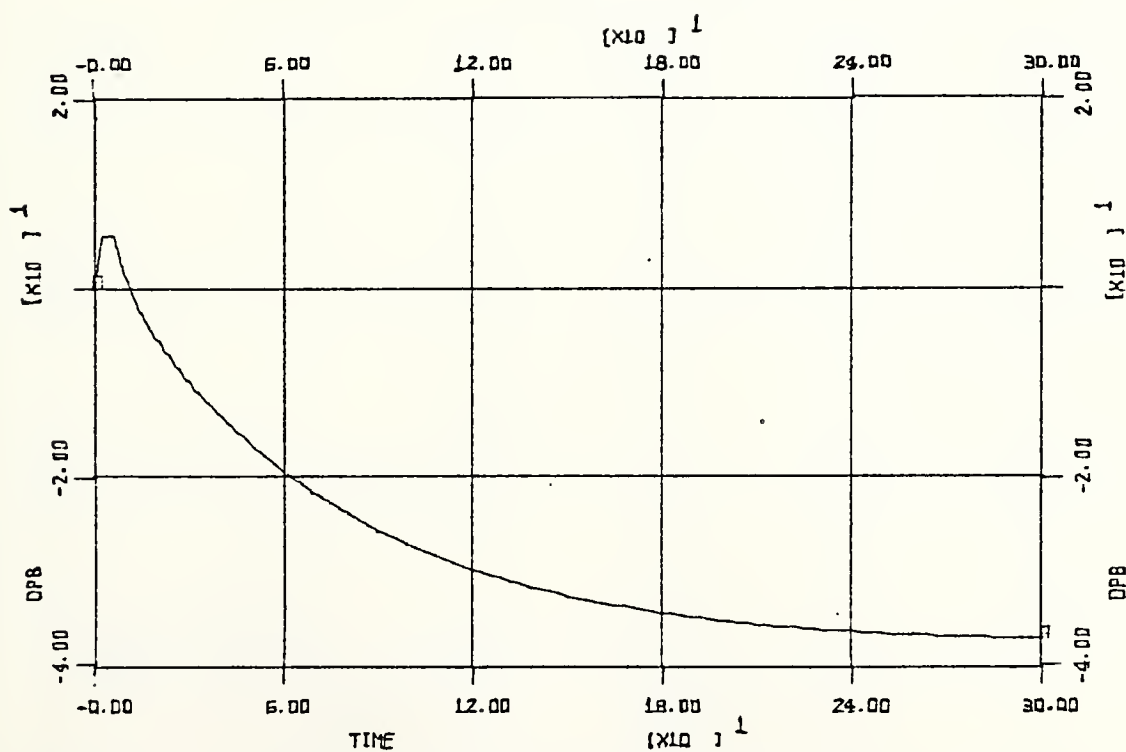
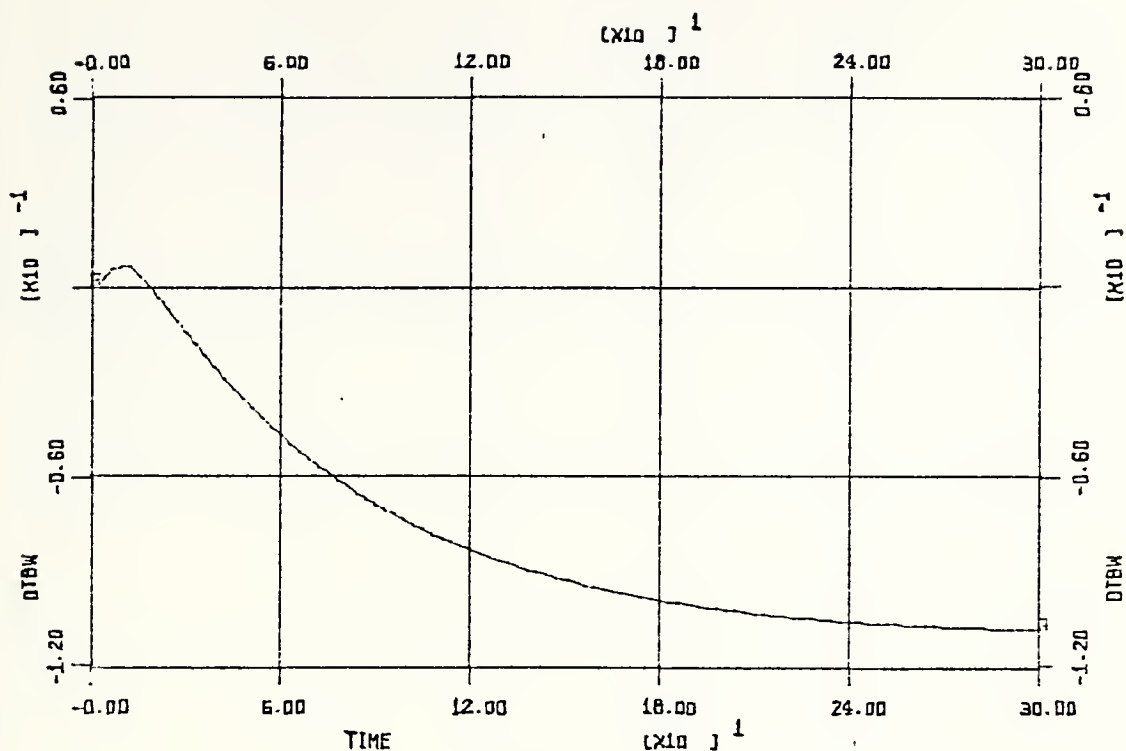


Figure 22 - RISER TUBE WALL TEMPERATURE ( $\Delta T_{BW}$ ) VS TIME AND STEAM DRUM PRESSURE ( $\Delta P_B$ ) VS TIME



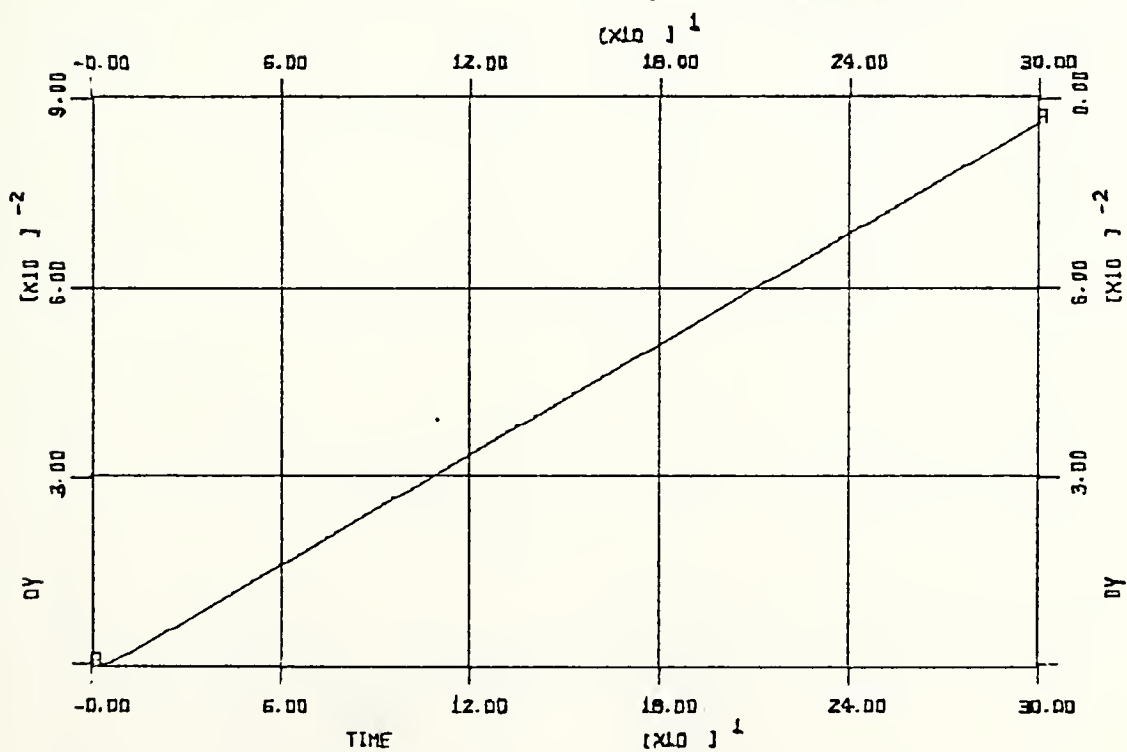
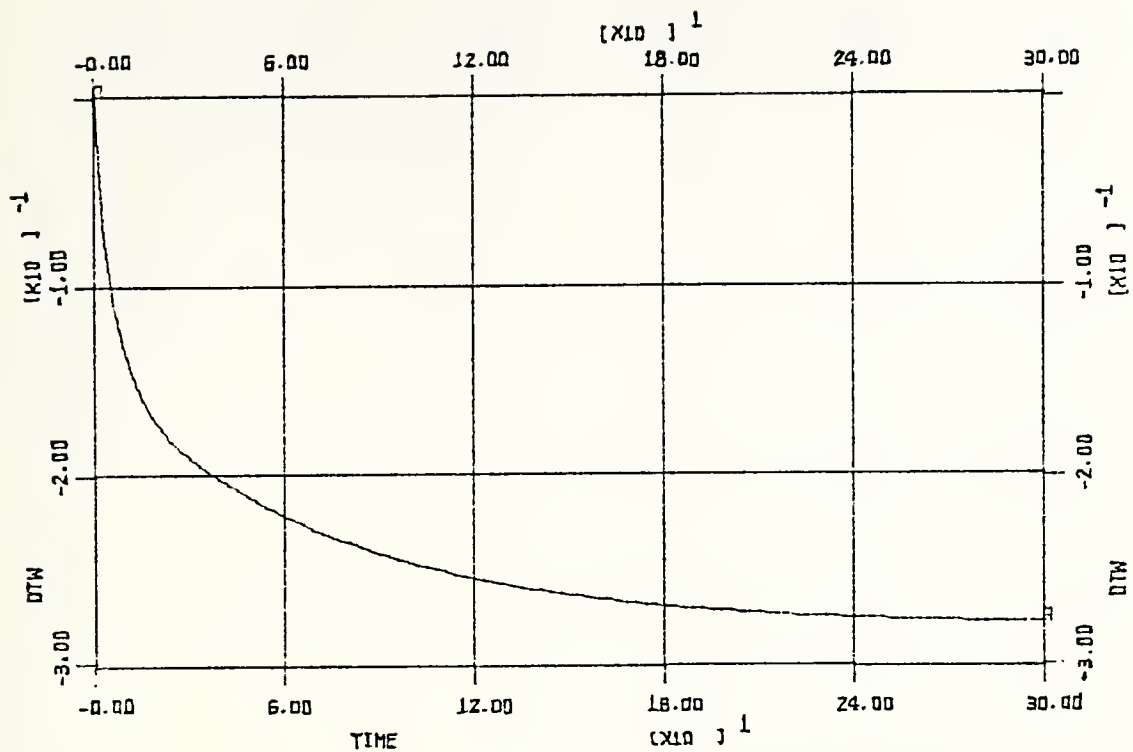


Figure 23 - STEAM DRUM LIQUID TEMPERATURE (  $\Delta T_W$  ) VS TIME  
AND STEAM DRUM WATER LEVEL (  $\Delta \gamma$  ) VS TIME



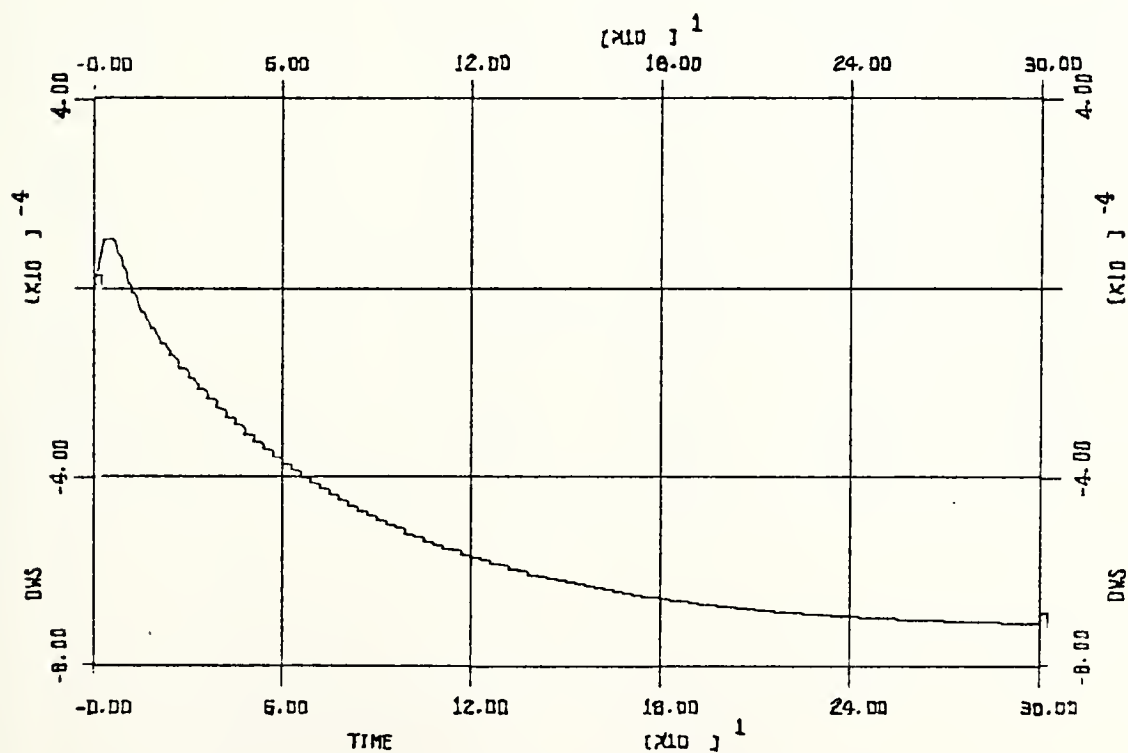
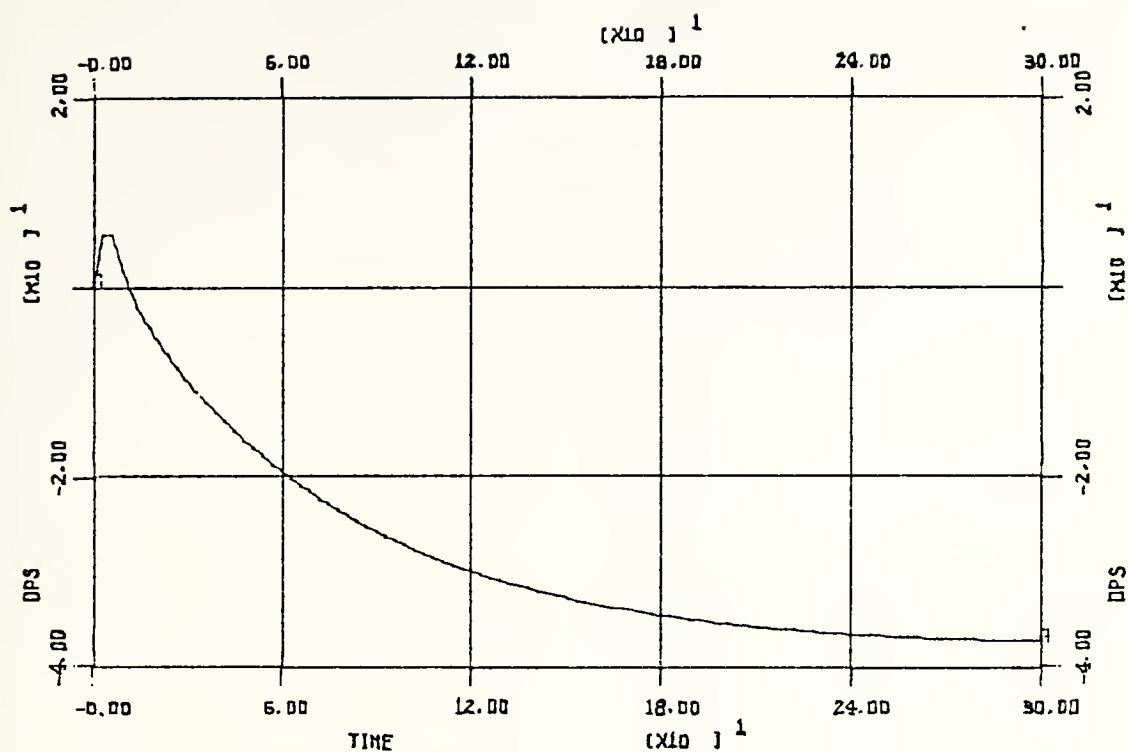


Figure 24 - SUPERHEATER OUTLET PRESSURE ( $\Delta P_s$ ) VS TIME  
AND STEAM FLOW RATE AT SUPERHEATER OUTLET ( $\Delta W_s$ ) VS TIME





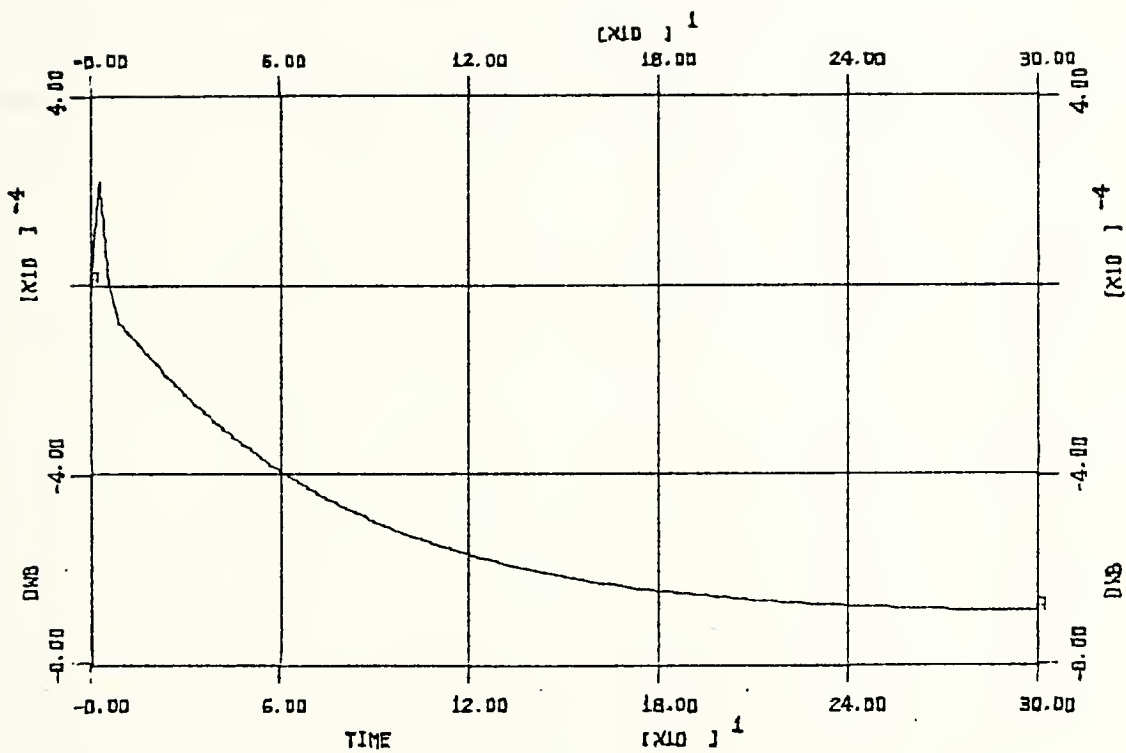


Figure 25 - STEAM MASS-FLOW RATE FROM STEAM DRUM TO  
SUPERHEATER (  $\Delta W_B$  ) VS TIME



#### IV. CONCLUSIONS

This analysis is only the initial step toward a better understanding of a naval steam generating plant both from the dynamic and from the control points of view. The analysis concerns a boiler system which is divided into four parts. The major difficulty in the analysis is the fact that the whole system is very complex and contains numerous variables which are extremely unwieldy to manipulate. Large numbers of relationships are non-linear so it is necessary to generate a linear form by the methods of perturbation theory. After setting up the state variable equations, the IBM simulation language CSMP-III was used to solve for the open loop transient response characteristics of the state variables due to variations in input variables. Throughout these calculations, all the variables appearing in the transient response curves are not absolute values. Rather, they represent a small incremental change from some particular steady state operating condition. Also, it is to be noted that trends in some of these results differ markedly in form from those appearing in Whalley (2). Future studies will be undertaken to determine the causes of these differences. Meanwhile, the benefit one may derive from this basic analysis is its use in the design and construction of a new type of boiler controller, that is, a multivariable control system.



# APPENDIX A

## LIST OF COEFFICIENTS A

$$a_1 = 1$$

$$a_2 = -1$$

$$a_3 = L_{SS}^A \bar{\rho}_s C_{ps}$$

$$a_4 = L_{SS}^A$$

$$a_5 = \bar{h}_S - \bar{h}_B$$

$$a_6 = -1$$

$$a_7 = -\bar{W}_S C_{ps}$$

$$a_8 = 1$$

$$a_9 = -1$$

$$a_{10} = M_{SS}^C$$

$$a_{11} = 1$$



$$a_{12} = \frac{0.6 K_{gs}}{\bar{W}_f^{0.4}} (\bar{T}_{gs} - \bar{T}_{sw})$$

$$a_{13} = K_{gs} \bar{W}_f^{0.6}$$

$$a_{14} = 2 K_{as} \frac{(\bar{W}_{ac} - \bar{W}_a)}{\bar{W}_{ac}^2}$$

$$a_{15} = 1$$

$$a_{16} = \frac{0.8 K_s}{\bar{W}_b^{0.2}} (\bar{T}_{sw} - \bar{T}_s)$$

$$a_{17} = K_S \bar{W}_B^{0.8}$$

$$a_{18} = \frac{1}{2 C_c}$$

$$a_{19} = (\bar{W}_f + \bar{W}_a)$$

$$a_{20} = (\bar{T}_C - \bar{T}_{gs})$$

$$a_{21} = (\bar{T}_C - \bar{T}_{gs})$$

$$a_{22} = -(\bar{W}_f + \bar{W}_a)$$

$$a_{23} = 2 f_s \frac{\bar{W}_b^2}{\bar{\rho}_b^2}$$

$$a_{24} = 1$$

$$a_{25} = -f_s \frac{\bar{W}_b^2}{\bar{\rho}_b^2}$$





$$a_{26} = 1$$

$$a_{27} = 0.5 A_r L_r \bar{\rho}^2 \left( \frac{1}{\bar{\rho}_w} - \frac{1}{\bar{\rho}_B} \right)$$

$$a_{28} = 0.5 A_r L_r \bar{X} K_B \left( \frac{\bar{\rho}^2}{\bar{\rho}_B^2} \right)$$

$$a_{29} = 0.5 (\bar{\rho} + \bar{\rho}_w) C$$

$$a_{30} = 1$$

$$a_{31} = -1$$

$$a_{32} = 1$$

$$a_{33} = -1$$

$$a_{34} = \frac{2\bar{W}}{g A_r^2 \rho} \left( \frac{f_r L_r}{D_r} + \frac{3}{2} \right)$$

$$a_{35} = \frac{L_r}{g A_r}$$

$$a_{36} = \left( \frac{1}{\bar{\rho}_B} - \frac{1}{\bar{\rho}_w} \right) \left[ \frac{\bar{W}^2}{g A_r^2} \left( \frac{f_r L_r}{D_r} + \frac{3}{2} \right) - \bar{\rho}^2 L_r \right]$$

$$a_{37} = \frac{-2\bar{W}_w}{g A_r^2 \bar{\rho}_w}$$

$$a_{38} = 1$$

$$a_{39} = -1$$



$$a_{40} = M_{B \ B}^C$$

$$a_{41} = 0.5 A_r L_r (\bar{\rho} + \bar{\rho}_w) K_c$$

$$a_{42} = 0.5 A_r L_r (\bar{\rho} + \bar{\rho}_w) \bar{h}_{fg}$$

$$a_{43} = -1$$

$$a_{44} = (\bar{h}_{WD} - \bar{h})$$

$$a_{45} = -\bar{W} K_C$$

$$a_{46} = -\bar{W} \bar{h}_{fg}$$

$$a_{47} = \bar{W}_w$$

$$a_{48} = 1$$

$$a_{49} = -K_{gB} \bar{W}_f^{0.6} - 4 K_r \bar{T}_{gB}^3$$

$$a_{50} = -K_{gB} \bar{W}_f^{0.6} - 4 K_r \bar{T}_{BW}^3$$

$$a_{51} = \frac{0.6 K_{gB} (\bar{T}_{gB} - \bar{T}_{BW})}{\bar{W}_f^{0.4}}$$

$$a_{52} = \frac{2 K_{ar} (\bar{W}_{ac} - \bar{W}_a)}{\bar{W}_{ac}^2}$$

$$a_{53} = 1$$



$$a_{54} = 3K_g (\bar{T}_{BW} - \bar{T}_B)^2$$

$$a_{55} = -3K_g (\bar{T}_{BW} - \bar{T}_B)^2$$

$$a_{56} = 1$$

$$a_{57} = -1$$

$$a_{58} = b_W = \frac{2W_W}{g\bar{\rho}_W A_D^2} \left( \frac{f_D L_D}{D_D} + 0.5 \right)$$

$$a_{59} = a_W = \frac{L_D}{g A_D}$$

$$a_{60} = \bar{V}_B K_B$$

$$a_{61} = -\bar{\rho}_B A$$

$$a_{62} = K_e$$

$$a_{63} = \bar{X}$$

$$a_{64} = \bar{W}$$

$$a_{65} = -1$$

$$a_{66} = (1 - \bar{X}) \bar{T}_B$$



$$a_{67} = (1-\bar{X}) \bar{W}$$

$$a_{68} = -\bar{W} \bar{T}_B$$

$$a_{69} = \bar{T}_i$$

$$a_{70} = -\bar{W}_W$$

$$a_{71} = -\bar{T}_W$$

$$a_{72} = -\bar{W}_e K_C$$

$$a_{73} = -\bar{h}_{BW} K_e$$

$$a_{74} = \bar{M}$$

$$a_{75} = A \bar{\rho}_W \bar{T}_W$$

$$a_{76} = -K_e$$

$$a_{77} = A \bar{\rho}_W$$

$$a_{88} = (1-\bar{X})$$

$$a_{89} = -\bar{W}$$

$$a_{90} = -1$$





$$a_{91} = -k_e$$

$$a_{92} = 1$$

$$a_{93} = 1$$

$$a_{94} = \frac{\partial f_1}{\partial x_v}$$

$$a_{95} = \frac{\partial f_1}{\partial p_s}$$

$$a_{96} = \frac{\partial f_1}{\partial T_s}$$

$$a_{97} = K_T$$

$$a_{98} = K_B$$

$$a_{99} = K_C$$

$$a_{100} = \frac{\partial f_2}{\partial T_s}$$

$$a_{101} = \frac{\partial f_2}{\partial p_s}$$

$$a_{102} = K_f$$



# APPENDIX B

## LIST OF COEFFICIENTS B

$$b_1 = \frac{a_1}{a_3}$$

$$b_2 = \frac{a_2}{a_3}$$

$$b_3 = \left( \frac{a_6 \cdot a_{17}}{a_4 \cdot a_{15}} + \frac{a_7}{a_4} \right)$$

$$b_4 = \left( \frac{a_5}{a_4} + \frac{a_6 \cdot a_{16}}{a_4 \cdot a_{15}} \right)$$

$$b_5 = \left( \frac{a_6 \cdot a_{17}}{a_4 \cdot a_{15}} \right)$$

$$b_6 = \left( \frac{a_{13} \cdot a_{162}}{a_{18}} - \frac{a_{12}}{a_{11}} + \frac{a_{20}}{a_{18}} \right) / \left( \frac{a_{13}}{a_{11}} - \frac{a_{22}}{a_{18}} \right)$$

$$b_7 = \left( \frac{a_{13}}{a_{11}} \right) / \left( \frac{a_{13}}{a_{11}} - \frac{a_{22}}{a_{18}} \right)$$

$$b_8 = \left( a_{14} - \frac{a_{21}}{a_{18}} \right) / \left( \frac{a_{13}}{a_{11}} - \frac{a_{22}}{a_{18}} \right)$$

$$b_9 = \left( \frac{a_{12}}{a_{11}} + \frac{a_{13} \cdot b_6}{a_{11}} \right)$$

$$b_{10} = \left( \frac{b_7 \cdot a_{13}}{a_{11}} - \frac{a_{13}}{a_{11}} \right)$$

$$b_{11} = \left( a_{14} - \frac{a_{13} \cdot b_8}{a_{11}} \right)$$



$$b_{12} = \frac{a_8 \cdot b_9}{a_{10}}$$

$$b_{13} = \left( \frac{a_9 \cdot b_{10}}{a_{10}} + \frac{a_9 \cdot a_{17}}{a_{10} \cdot a_{15}} \right)$$

$$b_{14} = \frac{a_9 \cdot b_{11}}{a_{10}}$$

$$b_{15} = \frac{a_9 \cdot a_{16}}{a_{10} \cdot a_{15}}$$

$$b_{16} = \frac{a_9 \cdot a_{17}}{a_{10} \cdot a_{15}}$$

$$b_{17} = \frac{a_{76} \cdot a_{97}}{a_{77}}$$

$$b_{18} = \frac{a_{88}}{a_{77}}$$

$$b_{19} = \frac{a_{89}}{a_{77}}$$

$$b_{20} = \frac{a_{90}}{a_{77}}$$

$$b_{21} = \frac{a_{91}}{a_{77}}$$

$$b_{22} = \frac{a_{92}}{a_{77}}$$

$$b_{23} = \left( \frac{a_{78} \cdot b_{17}}{a_{74}} + \frac{a_{97} \cdot a_{67}}{a_{74}} - \frac{a_{97} \cdot a_{73}}{a_{74}} + \frac{a_{72}}{a_{74}} \right)$$

$$b_{24} = \left( \frac{a_{66}}{a_{74}} - \frac{a_{75} \cdot b_{18}}{a_{74}} \right)$$

$$b_{25} = \left( \frac{a_{68}}{a_{74}} - \frac{a_{75} \cdot b_{19}}{a_{74}} \right)$$



$$b_{26} = \left( \frac{a_{71}}{a_{74}} - \frac{a_{75} \cdot b_{20}}{a_{74}} \right)$$

$$b_{27} = \left( \frac{a_{70} + a_{73}}{a_{74}} - \frac{a_{75} \cdot b_{21}}{a_{74}} \right)$$

$$b_{28} = \left( \frac{a_{69}}{a_{74}} - \frac{a_{75} \cdot b_{22}}{a_{74}} \right)$$

$$b_{29} = \left( \frac{a_{61} \cdot b_{17}}{a_{60}} - \frac{a_{62} \cdot a_{37}}{a_{60}} \right)$$

$$b_{30} = \left( \frac{a_{63}}{a_{60}} - \frac{a_{61} \cdot b_{18}}{a_{60}} \right)$$

$$b_{31} = \left( \frac{a_{64}}{a_{60}} - \frac{a_{61} \cdot b_{19}}{a_{60}} \right)$$

$$b_{32} = \left( \frac{a_{61} \cdot b_{20}}{a_{60}} \right)$$

$$b_{33} = \left( \frac{a_{62}}{a_{60}} - \frac{a_{61} \cdot b_{21}}{a_{60}} \right)$$

$$b_{34} = \frac{a_{61} \cdot b_{22}}{a_{60}}$$

$$b_{35} = \frac{a_{65}}{a_{60}}$$

$$b_{36} = \left( \frac{a_{45}}{a_{42}} - \frac{a_{41} \cdot b_{29}}{a_{42}} - \frac{a_{43} \cdot a_{55} \cdot a_{37}}{a_{42} \cdot a_{53}} \right)$$

$$b_{37} = \frac{a_{41} \cdot b_{30}}{a_{42}}$$

$$b_{38} = \left( \frac{a_{46}}{a_{42}} - \frac{a_{41} \cdot b_{31}}{a_{42}} \right)$$

$$b_{39} = \left( \frac{a_{44}}{a_{42}} + \frac{a_{41} \cdot b_{32}}{a_{42}} \right)$$





$$b_{40} = \left( \frac{a_{47}}{a_{42}} - \frac{a_{41} \cdot b_{33}}{a_{42}} \right)$$

$$b_{41} = \frac{a_{41} \cdot b_{34}}{a_{42}}$$

$$b_{42} = \frac{a_{41} \cdot b_{35}}{a_{42}}$$

$$b_{43} = \frac{a_{43} \cdot a_{54}}{a_{42} \cdot a_{53}}$$

$$b_{44} = \left( \frac{a_{27} \cdot b_{36}}{a_{29}} + \frac{a_{28} \cdot b_{29}}{a_{29}} \right)$$

$$b_{45} = \left( \frac{a_{27} \cdot b_{37}}{a_{29}} - \frac{a_{28} \cdot b_{30}}{a_{29}} + \frac{a_{31}}{a_{29}} \right)$$

$$b_{46} = \left( \frac{a_{27} \cdot b_{38}}{a_{29}} + \frac{a_{28} \cdot b_{31}}{a_{29}} \right)$$

$$b_{47} = \left( \frac{a_{28} \cdot b_{32}}{a_{29}} + \frac{a_{30}}{a_{29}} - \frac{a_{27} \cdot b_{39}}{a_{29}} \right)$$

$$b_{48} = \left( \frac{a_{27} \cdot b_{40}}{a_{29}} + \frac{a_{28} \cdot b_{33}}{a_{29}} \right)$$

$$b_{49} = \left( \frac{a_{28} \cdot b_{34}}{a_{29}} - \frac{a_{27} \cdot b_{41}}{a_{29}} \right)$$

$$b_{50} = \left( \frac{a_{27} \cdot b_{42}}{a_{29}} - \frac{a_{28} \cdot b_{35}}{a_{29}} \right)$$

$$b_{51} = \frac{a_{27} \cdot b_{43}}{a_{29}}$$

$$b_{52} = \frac{a_{58} + a_{59} \cdot b_{47}}{a_{57}}$$

$$b_{53} = \frac{a_{56} + a_{59} \cdot b_{44}}{a_{57}}$$



$$b_{54} = \frac{a_{59} \cdot b_{45}}{a_{57}}$$

$$b_{55} = \frac{a_{59} \cdot b_{46}}{a_{57}}$$

$$b_{56} = \frac{a_{59} \cdot b_{48}}{a_{57}}$$

$$b_{57} = \frac{a_{59} \cdot b_{49}}{a_{57}}$$

$$b_{58} = \frac{a_{59} \cdot b_{50}}{a_{57}}$$

$$b_{59} = \frac{a_{59} \cdot b_{51}}{a_{57}}$$

$$b_{60} = \left( \frac{a_{32} \cdot b_{52}}{a_{35}} - \frac{a_{37}}{a_{35}} \right)$$

$$b_{61} = \left( \frac{a_{33}}{a_{35}} - \frac{a_{32} \cdot b_{53}}{a_{35}} \right)$$

$$b_{62} = \left( \frac{a_{32} \cdot b_{54}}{a_{35}} - \frac{a_{34}}{a_{35}} \right)$$

$$b_{63} = \left( \frac{a_{32} \cdot b_{55}}{a_{35}} + \frac{a_{36}}{a_{35}} \right)$$

$$b_{64} = \frac{a_{32} \cdot b_{56}}{a_{35}}$$

$$b_{65} = \frac{a_{32} \cdot b_{57}}{a_{35}}$$

$$b_{66} = \frac{a_{32} \cdot b_{58}}{a_{35}}$$

$$b_{67} = \frac{a_{32} \cdot b_{59}}{a_{35}}$$



$$b_{68} = \left( \frac{a_{38} \cdot a_{50}}{a_{40} \cdot a_{48}} + \frac{a_{33} \cdot a_{54}}{a_{40} \cdot a_{53}} \right)$$

$$b_{69} = \left( \frac{a_{38} \cdot a_{51}}{a_{40} \cdot a_{48}} - \frac{a_{38} \cdot a_{49} \cdot b_6}{a_{40} \cdot a_{48}} \right)$$

$$b_{70} = \left( \frac{a_{38} \cdot a_{52}}{a_{40} \cdot a_{48}} + \frac{a_{38} \cdot a_{48} \cdot b_8}{a_{40} \cdot a_{48}} \right)$$

$$b_{71} = \frac{a_{38} \cdot a_{49} \cdot b_7}{a_{40} \cdot a_{48}}$$

$$b_{72} = \frac{a_{33} \cdot a_{55} \cdot a_{97}}{a_{40} \cdot a_{53}}$$

$$b_{73} = \left( \frac{a_{26}}{a_{23}} - \frac{a_{25} \cdot a_{98}}{a_{23}} \right)$$

$$b_{74} = \frac{a_{24} \cdot a_{100}}{a_{23}}$$

$$b_{75} = \frac{a_{24} \cdot a_{101}}{a_{23}}$$

$$b_{76} = \frac{a_{94}}{a_{93}}$$

$$b_{77} = \left( \frac{a_{95} \cdot a_{100}}{a_{93}} + \frac{a_{96}}{a_{93}} \right)$$

$$b_{78} = \frac{a_{95} \cdot a_{101}}{a_{93}}$$



## APPENDIX C

### C MATRIX COEFFICIENTS

$$C_{11} = (b_2 \cdot b_{78} - b_1 \cdot b_{75})$$

$$C_{12} = (b_2 \cdot b_{77} - b_1 \cdot b_{74})$$

$$C_{18} = b_1 \cdot b_{73}$$

$$C_{21} = b_4 \cdot b_{75}$$

$$C_{22} = (b_3 + b_4 \cdot b_{74})$$

$$C_{23} = -b_5$$

$$C_{28} = -b_4 \cdot b_{73}$$

$$C_{31} = -b_{15} \cdot b_{75}$$

$$C_{32} = -(b_{15} \cdot b_{74} + b_{16})$$

$$C_{33} = b_{13}$$

$$C_{38} = b_{15} \cdot b_{73}$$

$$C_{41} = b_{42} \cdot b_{75}$$

$$C_{42} = b_{42} \cdot b_{74}$$

$$C_{44} = b_{38}$$

$$C_{45} = -b_{37}$$

$$C_{46} = b_{39}$$

$$C_{47} = -b_{43}$$

$$C_{48} = (b_{36} - b_{42} \cdot b_{73})$$





$$C_{49} = b_{40}$$

$$C_{51} = -b_{66} \cdot b_{75}$$

$$C_{52} = -b_{66} \cdot b_{74}$$

$$C_{54} = -b_{63}$$

$$C_{55} = b_{62}$$

$$C_{56} = b_{60}$$

$$C_{57} = b_{67}$$

$$C_{58} = (b_{66} \cdot b_{73} + b_{61})$$

$$C_{59} = -b_{64}$$

$$C_{61} = -b_{50} \cdot b_{75}$$

$$C_{62} = -b_{50} \cdot b_{74}$$

$$C_{64} = -b_{46}$$

$$C_{65} = b_{45}$$

$$C_{66} = b_{47}$$

$$C_{67} = b_{51}$$

$$C_{68} = (b_{50} \cdot b_{73} - b_{44})$$

$$C_{69} = -b_{48}$$

$$C_{73} = -b_{71}$$

$$C_{77} = b_{68}$$

$$C_{78} = b_{72}$$

$$C_{81} = -b_{35} \cdot b_{75}$$

$$C_{82} = -b_{35} \cdot b_{74}$$

$$C_{84} = b_{31}$$



$$C_{85} = b_{30}$$

$$C_{86} = -b_{32}$$

$$C_{88} = (b_{29} + b_{35} \cdot b_{73})$$

$$C_{89} = b_{33}$$

$$C_{94} = b_{25}$$

$$C_{95} = b_{24}$$

$$C_{96} = b_{26}$$

$$C_{98} = b_{23}$$

$$C_{99} = b_{27}$$

$$C_{104} = b_{19}$$

$$C_{105} = b_{18}$$

$$C_{106} = b_{20}$$

$$C_{108} = -b_{17}$$

$$C_{109} = b_{21}$$



## APPENDIX D

### D MATRIX COEFFICIENTS

$$D_{11} = b_2 \cdot b_{76}$$

$$D_{32} = b_{12}$$

$$D_{33} = b_{14}$$

$$D_{44} = b_{41}$$

$$D_{54} = b_{65}$$

$$D_{64} = b_{49}$$

$$D_{72} = b_{69}$$

$$D_{73} = b_{70}$$

$$D_{84} = -b_{34}$$

$$D_{94} = b_{28}$$

$$D_{104} = b_{22}$$



# APPENDIX E

## LIST OF CONSTANTS

$L_S$	= 45.0
$A_S$	= 0.239
$\bar{\rho}_S$	= 0.5
$C_{ps}$	= 0.53
$\bar{h}_S$	= 1390.0
$\bar{h}_B$	= 1204.9
$\bar{W}_S$	= 4.0
$M_S$	= 1900.0
$C_S$	= 0.23
$K_{gs}$	= 0.94633
$\bar{W}_f$	= 0.32
$\bar{T}_{gs}$	= 2800.0
$\bar{T}_{sw}$	= 1250.0
$K_{as}$	= 0.0
$\bar{W}_{ac}$	= 7.0
$\bar{W}_a$	= 5.3
$K_S$	= 4.8848
$\bar{T}_S$	= 1200.0





$$\begin{aligned}
\bar{W}_B &= 4.0 \\
C_C &= 0.37 \\
\bar{T}_C &= 3200.0 \\
f_S &= 68.22 \\
\bar{\rho}_B &= 0.758 \\
A_r &= 2.95 \\
L_r &= 10.0 \\
\bar{\rho} &= 16.52 \\
\bar{\rho}_w &= 52.6 \\
\bar{X} &= 0.031933 \\
K_B &= 0.00001 \\
C &= 0.007637 \\
\bar{W} &= 178.97 \\
f_r &= 0.76095 \\
D_r &= 0.146 \\
g &= 32.2 \\
\bar{W}_w &= 178.97 \\
M_B &= 12000.0 \\
C_B &= 0.28 \\
K_C &= 0.00406 \\
\bar{h}_{fg} &= 390.0 \\
\bar{h}_{wD} &= 0.0 \\
\bar{h} &= 430.39 \\
K_{gB} &= 3.6632 \\
K_r &= 7.5E-10 \\
\bar{T}_{gB} &= 3000.0
\end{aligned}$$



$$\bar{T}_{bw} = 930.0$$

$$K_{ar} = 0.0$$

$$K_g = 0.059806$$

$$\bar{T}_B = 890.0$$

$$f_D = 2.7014$$

$$L_D = 10.0$$

$$D_D = 0.3$$

$$A_D = 0.426$$

$$\bar{V}_B = 40.0$$

$$A = 26.4$$

$$K_e = -0.39157$$

$$\bar{T}_i = 740.0$$

$$\bar{T}_W = 894.38$$

$$w_e = -1.7742$$

$$\bar{h}_{BW} = 405.0$$

$$M = 1680.0$$

$$K_T = 0.003$$

$$K_f = 500.0$$



# APPENDIX F

## FORTRAN IV PROGRAM

\$JOB

IMPLICIT REAL\*4 (A-Z)

LS=45.0

AS=0.239

RHOS=0.5

CPS=0.53

FS=1390.0

FB=1204.9

WS=4.0

HS=1900.0

CS=0.23

KGS=0.94653

WF=0.32

TGS=2300.0

TSW=1250.0

KAS=0.0

WAC=7.0

WA=5.3

KS=4.6942

TS=1200.0

WB=4.0

CC=0.37

IC=3200.0

FS=63.22

RHOB=0.758

AR=2.95

LR=10.0

RHO=10.52

RHON=52.6

X=0.031933

KB=0.00001

C=0.007637

W=178.97

FR=0.75095

OR=0.146

G=22.2

WW=175.97

MB=12000.0

CB=0.23

KC=0.00400

HFG=190.0

HFD=0.0

H=430.33

KGB=3.6632

KR=7.5F=10

TGB=3000.0

TB=930.0

KAR=0.0

KG=0.059206

IB=370.0

FD=2.7014

LD=10.0

DD=0.3

AD=0.426

VB=40.0

A=20.+

KF=-0.36157

TL=740.0

IN=394.38

WE=-1.7742



```

PBW=405.0
M=1680.0
KT=0.003
KF=500.0
A1=1.0
A2=-1.0
A3=LS*AS
A4=LS*AS*RHDS*CPS
A5=FS-HH
A6=-1.0
A7=-AS*CPS
A8=1.0
A9=-1.0
A10=MS*CS
A11=1.0
A12=(0.6*KS)/(WF**0.4)*(TGS-TSW)
A13=KGS*WF**0.6
A14=(2.5*KAS/WAC**2)*(WAC-WA)
A15=1.0
A16=(0.9*KS/WF**0.2)*(TSW-TS)
A17=KS*WB**0.2
A18=1.0/(2.0*CC)
A19=WF+WA
A20=TC-TGS
A21=A20
A22=-(WF+WA)
A23=2.0*FS*WB/RHDB
A24=1.0
A25=-(FS*WB**2/RHDB**2)
A26=1.0
A27=0.5*A*LR*REC**2*(1.0/RHDB-1.0/RHDB)
A28=0.5*AR*LR*X*KB*(RHO**2/RHDB**2)
A29=0.5*CR*(RHO+RECW)
A30=1.0
A31=-1.0
A32=1.0
A33=-1.0
A34=2.0*W*(FR*LR/DR+1.5)/(G*AR**2*RHO)
A35=LR/(-G*AR)
A36=(1.0/RHDB-1.0/RECW)*((W**2/(G*AR**2))*(FR*LR/DR+1.5)-
1*(RHO**2*LR))
A37=-2.0*W*W/(G*AR**2*(F+EW))
A38=1.0
A39=-1.0
A40=WB*CB
A41=0.5*AR*LR*KC*(RHO+RHO4)
A42=0.5*AR*LR*REC*(REC+RHOW)
A43=-1.0
A44=HWD-H
A45=-W*KC
A46=-W*REC
A47=WA
A48=1.0
A49=(-KGB*WF**0.6)-(4.0*KR*TGB**2)
A50=(-KGB*WF**0.6)-(4.0*KR*TBW**2)
A51=0.6*KGB*(TGB-TBW)/WF**0.4
A52=2.0*KAR*(WAC-WA)/WAC**2
A53=1.0
A54=3.0*KG*(TBW-TB)**2.0
A55=-A54
A56=1.0
A57=-1.0
A58=2.0*AW*(FO*LC/LC+0.5)/(AD**2.0*G*RECW)
A59=LD/(G*AD)
A60=VB*KE
A61=-REC+FA
A62=KF
A63=X
A64=W
A65=-1.0

```





```

A66=TB*(1.0-X)
A67=W*(1.0-X)
A68=-W*TB
A69=TI
A70=-WW
A71=-TW
A72=-WE*KC
A73=-HBW*KE
A74=M
A75=A*RHCW*TW
A76=-KE
A77=A*RHGW
A88=1.0-X
A89=-W
A90=-1.0
A91=-KE
A92=1.0
A93=1.0
A94=0.42
A95=0.000019
A96=-0.000005
A97=KT
A98=K3
A99=K6
A100=240.0
A101=95000.0
A102=KF
B1=A1/A3
B2=A2/A3
B3=(A6*A17)/((A4*A15)+(A7/A4))
B4=(A6*A15)/((A4*A15)+(A5/A4))
B5=(A6*A17)/((A4*A15))
B6=((A19*A102/A18)-(A12/A11)+(A20/A18))/((A13/A11)-(A22/A18))
B7=(A13/A11)/((A13/A11)-(A22/A18))
B8=(A14-(A21/A15))/((A13/A11)-(A22/A18))
B9=(A12/A11)+(A13*B6/A11)
B10=(A7*A15/A11)-(A13/A11)
B11=A14-(A8*A13/A11)
B12=A8*B9/A10
B13=(A8*B10/A10)+((A9*A17)/(A10*A15))
B14=A9*B11/A10
B15=(A9*A16)/(A10*A15)
B16=(A9*A17)/(A10*A15)
B17=A76*A97/A77
B18=A88/A77
B19=A95/A77
B20=A90/A77
B21=A91/A77
B22=A92/A77
B23=(A75*B17/A74)+(A97*A67/A74)-(A97*A73/A74)+(A72/A74)
B24=(A65/A74)-(A75*B18/A74)
B25=(A68/A74)-(A75*B19/A74)
B26=(A71/A74)-(A75*B20/A74)
B27=(A70+A73)/A74-(A75*B21/A74)
B28=(A69/A74)-(A75*B22/A74)
B29=(A61*B17/A60)-(A62*A97/A60)
B30=(A63/A60)-(A61*B18/A60)
B31=(A64/A60)-(A61*B19/A60)
B32=A61*B20/A60
B33=(A62/A60)-(A61*B21/A60)
B34=A61*B22/A60
B35=A65/A60
B36=(A46/A42)-(A41*B23/A42)-(A43*A55*A97/A42/A53)
B37=A41*B24/A42
B38=(A46/A42)-(A41*B31/A42)
B39=(A44/A42)+(A41*B22/A42)
B40=(A47/A42)-(A41*B25/A42)
B41=A41/A42*B34
B42=A41*B35/A42

```



$B43 = (A43 * A54) / (A42 * A53)$   
 $B44 = (A27 * B35 / A29) + (A27 * B29 / A29)$   
 $B45 = (A27 * B37 / A29) - (A28 * B30 / A29) + (A31 / A29)$   
 $B46 = (A27 * B33 / A29) + (A28 * B31 / A29)$   
 $B47 = (A23 * B32 / A29) + (A30 / A29) - (A27 * B36 / A29)$   
 $B48 = (A27 * B40 / A29) + (A28 * B38 / A29)$   
 $B49 = (A28 * B34 / A29) - (A27 * B41 / A29)$   
 $B50 = (A27 * B42 / A29) - (A28 * B35 / A29)$   
 $B51 = A27 * B43 / A29$   
 $B52 = (A53 / A57) + (A59 * B47 / A57)$   
 $B53 = (A56 / A57) + (A59 * B44 / A57)$   
 $B54 = A59 * B45 / A57$   
 $B55 = A59 * B46 / A57$   
 $B56 = A59 * B48 / A57$   
 $B57 = A59 * B49 / A57$   
 $B58 = A59 * B50 / A57$   
 $B59 = A59 * B51 / A57$   
 $B60 = (A32 * B52 / A35) - (A27 / A35)$   
 $B61 = (A32 / A35) - (A32 * B52 / A35)$   
 $B62 = (A32 * B54 / A35) - (A34 / A35)$   
 $B63 = (A32 * B55 / A35) + (A36 / A35)$   
 $B64 = A32 * B56 / A35$   
 $B65 = A32 * B57 / A35$   
 $B66 = A32 * B58 / A35$   
 $B67 = A32 * B59 / A35$   
 $B68 = (A33 * A50) / (A40 * A43) + (A39 * A54) / (A40 * A53)$   
 $B69 = (A38 * A51) / (A40 * A43) - (A38 * A45 * B6) / (A40 * A43)$   
 $B70 = (A38 * A52) / (A40 * A43) + (A38 * A49 * B8) / (A40 * A43)$   
 $B71 = (A33 * A47 * B7) / (A40 * A43)$   
 $B72 = (A39 * A55 * A97) / (A40 * A53)$   
 $B73 = (A26 / A23) - (A25 * A99 / A23)$   
 $B74 = A24 * A100 / A23$   
 $B75 = A24 * A101 / A23$   
 $B76 = A94 / A93$   
 $B77 = (A95 * A100 / A93) + (A96 / A93)$   
 $B78 = A95 * A101 / A93$   
 $C11 = (B2 * E78) - (B1 * B75)$   
 $C12 = (B2 * B77) - (B1 * B74)$   
 $C13 = B1 * B73$   
 $C21 = B4 * B75$   
 $C22 = B3 + (B4 * B74)$   
 $C23 = -B5$   
 $C28 = -B4 * B73$   
 $C31 = -B15 * B75$   
 $C32 = -(B15 * B74) - B16$   
 $C33 = B13$   
 $C38 = B15 * B73$   
 $C41 = B42 * E75$   
 $C42 = B42 * B74$   
 $C44 = B33$   
 $C45 = -B37$   
 $C46 = B39$   
 $C47 = -B43$   
 $C48 = B36 - (B42 * E73)$   
 $C49 = B49$   
 $C51 = -B65 * B75$   
 $C52 = -B66 * B74$   
 $C54 = -B63$   
 $C55 = B62$   
 $C56 = B60$   
 $C57 = B67$   
 $C58 = (B66 * B73) + B61$   
 $C59 = -B64$   
 $C61 = -B50 * B75$   
 $C62 = -B50 * B74$   
 $C64 = -B48$   
 $C65 = B45$   
 $C66 = B47$   
 $C67 = B51$   
 $C68 = (B50 * B73) - B44$   
 $C69 = -B43$   
 $C73 = -B71$



```

C77=B69
C78=B72
C81=-B55*B75
C82=-B35*B74
C84=B31
C85=B30
C86=-B32
C88=B29+(B35*B73)
C89=B33
C94=B25
C95=B24
C96=B26
C98=B23
C99=B27
C104=B15
C105=B18
C106=B20
C108=-B17
C109=B21
D11=B2*B76
D32=B12
D33=B14
D44=B41
D54=B65
D64=B49
D72=B35
D73=B70
D84=-B34
D94=B28
D104=B22
WRITE (2,100) A100,A101,D73,B74,B75,B76,B77,B78
100 FORMAT(1H,8X,5HA100=,E15.3,5X,5HA101=,E15.8//,
19X,5HB72=,E15.8,5X,5HB74=,E15.8,5X,5HB75=,E15.8//,
29X,5HB76=,E15.8,5X,5HB77=,E15.8,5X,5HB78=,E15.8//)
WRITE (2,100) C11,C12,C13,C14,C15,C16,C17,C18,C19,C20,C21,C22,C23,C24,C25,C26,C27,C28,C29,C30,C31,C32,C33,C34,C35,C36,C37,C38,C39,C40,C41,C42,C43,C44,C45,C46,C47,C48,C49,C50,C51,C52,C53,C54,C55,C56,C57,C58,C59,C60,C61,C62,C63,C64
200 FORMAT(1H,8X,5HC11=,E15.3,5X,5HC12=,E15.3,5X,5HC13=,E15.8//,
19X,5HC21=,E15.8,5X,5HC22=,E15.8,5X,5HC23=,E15.8//,
29X,5HC31=,E15.8,5X,5HC32=,E15.8,5X,5HC33=,E15.8//,
39X,5HC33=,E15.8,5X,5HC34=,E15.8,5X,5HC35=,E15.8//,
49X,5HC42=,E15.8,5X,5HC43=,E15.8,5X,5HC44=,E15.8//,
59X,5HC45=,E15.8,5X,5HC46=,E15.8,5X,5HC47=,E15.8//,
69X,5HC49=,E15.8,5X,5HC51=,E15.8,5X,5HC52=,E15.8//,
79X,5HC54=,E15.8,5X,5HC55=,E15.8,5X,5HC56=,E15.8//,
89X,5HC57=,E15.8,5X,5HC58=,E15.8,5X,5HC59=,E15.8//,
99X,5HC61=,E15.8,5X,5HC62=,E15.8,5X,5HC63=,E15.8//,
WRITE (2,200) C65,C66,C67,C68,C69,C70,C71,C72,C73,C74,C75,C76,C77,C78,C79,C80,C81,C82,C83,C84,C85,C86,C87,C88,C89,C90,C91,C92,C93,C94,C95,C96,C97,C98,C99,C100,C101,C102,C103,C104,C105,C106,C107,C108,C109
300 FORMAT(1H,8X,5HC65=,E15.8,5X,5HC66=,E15.8,5X,5HC67=,E15.8//,
19X,5HC68=,E15.8,5X,5HC69=,E15.8,5X,5HC70=,E15.8//,
29X,5HC77=,E15.8,5X,5HC78=,E15.8,5X,5HC79=,E15.8//,
39X,5HC82=,E15.8,5X,5HC83=,E15.8,5X,5HC84=,E15.8//,
49X,5HC86=,E15.8,5X,5HC87=,E15.8,5X,5HC88=,E15.8//,
59X,5HC94=,E15.8,5X,5HC95=,E15.8,5X,5HC96=,E15.8//,
69X,5HC98=,E15.8,5X,5HC99=,E15.8,5X,5HC100=,E15.8//,
79X,5HC106=,E15.8,5X,5HC107=,E15.8,5X,5HC108=,E15.8//,
89X,5HC109=,E15.8//)
WRITE (4,400) D11,D32,C33,D44,D54,D64,D72,D73,D84,D94,D104
400 FORMAT(1H,8X,5HD11=,E15.8,5X,5HD32=,E15.3,5X,5HD33=,E15.8//,
19X,5HD44=,E15.8,5X,5HD54=,E15.8,5X,5HD64=,E15.8//,
29X,5HD72=,E15.8,5X,5HD73=,E15.8,5X,5HD84=,E15.8//,
39X,5HD94=,E15.8,5X,5HD104=,E15.8//)
STOP
END

```



# APPENDIX G

## FORTRAN IV PROGRAM OUTPUT

A100= 0.24000000E 03	A101= 0.95000000E 05	
B73 = 0.14152740E-02	B74 = 0.33333340E 00	B75 = 0.13194440E 03
B76 = 0.+20000000E 00	B77 = 0.45545540E-02	B78 = 0.16045990E 01
C11 =-0.12436020E 02	C12 =-0.31416850E-01	C13 = 0.13159210E-03
C21 = 0.17133750E 04	C22 =-0.16096850E 01	C23 = 0.51956340E 01
C28 =-0.18385510E-01	C31 = 0.44709970E 02	C32 = 0.14883690E 00
C33 =-0.24865910E-01	C33 =-0.47957190E-03	C41 =-0.34339410E 01
C42 =-0.86752210E-02	C44 =-0.47662320E 01	C45 =-0.11941460E-02
C46 =-0.70738900E-05	C47 = 0.72198050E-03	C48 = 0.27080380E-05
C49 = 0.10494110E-01	C51 =-0.49083270E 05	C52 =-0.12402500E 04
C54 =-0.63842140E 03	C55 =-0.15757540E 02	C56 = 0.93258380E 03
C57 = 0.99146950E 02	C58 = 0.40967170E 00	C59 = 0.14977520E 04
C61 =-0.70394060E 05	C62 =-0.17910070E 03	C64 =-0.98256370E 05
C65 =-0.28442070E 02	C66 =-0.99339400E 01	C67 = 0.14317490E 02
C68 = 0.59159390E-01	C69 = 0.21628560E 03	C73 = 0.25403440E-02
C77 =-0.86705500E-01	C78 = 0.25631110E-03	C81 = 0.32986120E 06
C82 = 0.83333390E 03	C84 = 0.44097750E 05	C85 = 0.11470370E 03
C86 =-0.56026590E 02	C88 =-0.64373300E 00	C89 =-0.96481320E 03
C94 = 0.46855270E 00	C95 =-0.25236600E-02	C96 =-0.29902320E-06
C98 = 0.65586110E-03	C99 =-0.22059300E 00	C104=-0.12888150E 00
C105= 0.69713330E-03	C106=-0.72012910E-03	C108=-0.84594270E-06
C109= 0.28198050E-03		
D11 =-0.39051580E-01	D32 = 0.34094850E 01	D33 = 0.69783800E-01
D44 =-0.37504610E-03	D54 =-0.53618420E 02	D64 =-0.77428660E 01
D72 = 0.73057530E 01	D75 = 0.15741700E 01	D94 = 0.36026590E 02
D94 =-0.91892420E-01	D104= 0.72012910E-03	





# APPENDIX H

## CSMP - III PROGRAM

\$\$\$CONTINUOUS SYSTEM MODELING PROGRAM \*\*\* VIMS TRANSLATION OUTPUTS

INITIAL

CONSTANT A100=0.2400E+03,A101=0.5570E+05  
 CONSTANT B73=0.1415E-02,B74=0.2323E+02,B75=0.1319E+03  
 CONSTANT B76=0.4200E+00,B77=0.4555E-02,B78=0.1205E+01  
 CONSTANT C11=-0.1343E+02,C12=-0.5142E-01,C13=0.1316E-03  
 CONSTANT C21=0.1714E+04,C22=-0.1609E+01,C23=0.5193E+01  
 CONSTANT C24=-0.1338E-01,C31=0.4271E+02,C32=0.1458E+00  
 CONSTANT C33=-0.2486E-01,C34=-0.4736E-03,C41=-0.3434E+01  
 CONSTANT C42=-0.8875E-02,C44=-0.4756E+01,C45=-0.1194E-02  
 CONSTANT C46=-0.7370E-05,C47=0.7219E-05,C48=0.2727E-05  
 CONSTANT C49=0.1049E-01,C51=-0.4609E+06,C52=-0.1240E+04  
 CONSTANT C54=-0.6338E+06,C55=-0.1575E+02,C56=0.9225E+03  
 CONSTANT C57=0.9914E+02,C58=0.4053E+01,C59=0.1497E+04  
 CONSTANT C61=0.7039E+05,C62=-0.1791E+03,C64=-0.9325E+05  
 CONSTANT C65=-0.2844E+02,C66=-0.5823E+01,C67=0.1423E+02  
 CONSTANT C68=0.5910E-01,C69=0.2162E+03,C72=0.2540E-02  
 CONSTANT C77=-0.8670E-01,C78=0.2562E-03,C81=0.3290E+06  
 CONSTANT C82=0.8335E+03,C84=0.4414E+03,C85=0.1147E+03  
 CONSTANT C86=-0.3603E+02,C88=-0.6437E+03,C89=-0.9648E+02  
 CONSTANT C94=0.4665E+00,C95=-0.2824E-02,C96=-0.2930E-06  
 CONSTANT C98=0.6558E-03,C99=-0.2204E+00,C104=-0.1253E+00  
 CONSTANT C105=0.8971E-03,C106=-0.7201E-03,C109=-0.8459E-06  
 CONSTANT C109=0.2817E-05  
 CONSTANT D11=-0.3405E-01,D32=0.3403E+01,D33=0.6973E-01  
 CONSTANT D44=-0.3750E-03,D54=-0.5261E+02,D64=-0.7742E+01  
 CONSTANT D72=0.7535E+01,D75=0.1574E+01,D76=0.2360E+02  
 CONSTANT D94=-0.9133E-01,D104=0.7201E-05  
 CONSTANT IC1=0.0,IC2=0.0,IC3=0.0,IC4=0.0,IC5=0.0  
 CONSTANT IC6=0.0,IC7=0.0,IC8=0.0,IC9=0.0,IC10=0.0  
 CONSTANT DXV=0.05,DVA=0.0,DWF=0.0,DWT=0.0

DYNAMIC

DRHSD=C11\*DRHUS+C12\*DTs+C13\*DPB+D11\*DXV  
 DRHOS=INTGRL(IC1,DRHSD)  
 DTSD=C21\*DRHOS+C22\*DTs+C23\*DTSN+C24\*DPB  
 DTs=INTGRL(IC2,DTSD)  
 DTSDW=C31\*DRHOS+C32\*DTs+C33\*DTSN+C34\*DPB+D32\*DWF+D33\*DWA  
 DTSW=INTGRL(IC3,DTSDW)  
 DXD=C41\*DRHOS+C42\*DTs+C44\*DX+C45\*DW+C46\*DWX+C47\*DTPM ...  
 +C48\*DPB+C49\*DTW+C44\*DW  
 DX=INTGRL(IC4,DXD)  
 DW=D=C51\*DRHOS+C52\*DTs+C54\*DX+C55\*DW+C56\*DWX+C57\*DTPM ...  
 +C58\*DPB+C59\*DTW+C55\*DW  
 DW=INTGRL(IC5,DW)  
 DWWD=C61\*DRHOS+C62\*DTs+C64\*DX+C65\*DW+C66\*DWX+C67\*DT64 ...  
 +C68\*DPB+C69\*DTW+C64\*DW  
 DWW=INTGRL(IC6,DWW)  
 DTBW=C73\*DTs+C77\*DTLW+C78\*DPB+D72\*DWX+D73\*DWA  
 DTBW=INTGRL(IC7,DTBW)  
 DPBD=C81\*DRHOS+C82\*DTs+C84\*DX+C85\*DW+C86\*DWX+C88\*DPB+C89\*DTW ...  
 +D84\*DW  
 DPB=INTGRL(IC8,DPBD)  
 DTWD=C94\*DX+C95\*DW+C96\*DWX+C98\*DPB+C99\*DTW+D94\*DW  
 DTW=INTGRL(IC9,DTWD)  
 DYD=C104\*DX+C105\*DW+C106\*DWX+C108\*DPB+C109\*DTW+D104\*DW  
 DY=INTGRL(IC10,DYD)  
 DPS=A100\*DTs+A101\*DPB  
 DVB=B73\*DPB-B74\*DTs-B75\*DRHOS  
 DWS=B76\*DXV+B77\*DTs+B78\*DPB



```

PAGE HEIGHT=3,WIDTH=5
PAGE XYPLOT
OUTPUT TIME,DRHOS
LABEL DRHOS VS TIME
OUTPUT TIME,DTS
LABEL DTS VS TIME
OUTPUT TIME,DTSW
LABEL DTSW VS TIME
OUTPUT TIME,DX
LABEL DX VS TIME
OUTPUT TIME,DX
LABEL DW VS TIME
OUTPUT TIME,DW
LABEL DW VS TIME
OUTPUT TIME,DTBW
LABEL DTBW VS TIME
OUTPUT TIME,DPB
LABEL DPB VS TIME
OUTPUT TIME,DTW
LABEL DTW VS TIME
OUTPUT TIME,DY
LABEL DY VS TIME
OUTPUT TIME,DPS
LABEL DPS VS TIME
OUTPUT TIME,DWS
LABEL DWS VS TIME
OUTPUT TIME,DWB
LABEL DWB VS TIME
TIMER FINTIME=300.0,OUTDEL=3.0,PRDEL=0.0
END
CONSTANT DWF=0.032,DXV=0.0,DWA=0.0,DWT=0.0
END
CONSTANT DWI=0.4,DWA=0.0,DWF=0.0,DXV=0.0
END
STOP

```



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